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SUMMARY

THE USE OF TEMPERATURE AS A VARIABLE IN HYDRAULICS

by

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Use of Bernoulli (or Euler) equation in Hydraulics is restricted to flows which can be considered frictionless and in which there is no external work transfer.

Flow of a liquid causes changes in the thermodynamic state of the liquid - these changes being inevitably irreversible. It is clear therefore that a complete description of liquid flow processes can only be obtained by using the more general equations of thermodynamics. The main obstacle to this procedure has been the difficulty of measuring the small temperature differences encountered in liquid flow and on which the thermodynamic equations are dependent.

Thus research was aimed first at developing simple methods and techniques for measuring small temperature differences such as those occurring in hydraulic (or liquid) flow systems; and at surmounting the

various difficulties inherent in such measurements. As a result it is now in most cases possible to obtain an accuracy of $\pm$ 1 m C^{o} in the measurement of temperature difference across a hydraulic flow system (1 millidegree = 0.001 C^{o}). Under favourable conditions an accuracy of $\pm$ 0.3 m C^{o} has been attained. The apparatus used for such measurements is cheap and very simple.

A suitable equation for use in the thermodynamic treatment of liquid flow is the Steady Flow Energy Equation. It was found necessary, however, to recast this equation in a special form for convenient application to liquid flow. This necessitated introduction of certain thermodynamic properties (e.g. $(\partial h/\partial P)_T$). The values of these properties for water were evaluated and presented in tabular form for the ranges 1-50°C and 1-200 atmospheres. Examples of the order of magnitude of these properties for some other common liquids are also shown.

In contrast to Bernoulli and Euler equations this version of the Steady Flow Energy Equation depends on both the temperature and pressure as variables. The equation has been used in the analysis of a number of hydraulic flow systems of varied nature with very satisfactory results.

Research/

Research was conducted to show the practicability and advantages of using the temperature as a variable in two imporant fields of hydraulic practice, namely the measurement of efficiency and of the rate of flow. The development of the techniques of temperature measurement - referred to above - rendered practicable the application of the thermodynamic methods of measuring hydraulic efficiency to machines operating at heads as low as 100 feet - lower than hitherto possible. The development of these techniques of thermometry also rendered practicable the use of novel and simple thermometric methods for measuring the rate of flow.

In conclusion it is hoped that the satisfactory results obtained in this research will help to inspire more confidence in the use of temperature as a variable in Hydraulics and stimulate interest in Thermohydraulics - the thermodynamic treatment of liquid flow.

December, 1964.

PH.D. THESIS

THE USE OF TEMPERATURE AS A VARIABLE IN HYDRAULICS

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P R E F A C E

DEFINITION OF THE TERM HYDRAULICS AS USED IN THIS THESIS

Hydraulics in the original sense meant the study of water and its application, or briefly 'water engineering.' As the engineers interest in other fluids increased, the coverage of Hydraulics was extended to the study of other liquids and, on occasions, gases. However it was soon realised that liquids and gases have many divergent properties and it is inconvenient to deal with both under the same heading. For this reason the use of the term Hydraulics was then confined to the study of liquids and liquid flow. In this thesis, Hydraulics will be taken to cover this latter meaning.

Limitation of the Practical research work undertaken

For practical reasons all experimental work was done only on water at room temperatures. However there is no reason why the conclusions drawn may not be applicable to other liquids satisfying the pure substance condition; or to water and to these liquids at temperatures different from room temperature provided that heat transfer to the

surroundings is accounted for. The necessary thermodynamic properties of water are evaluated and tabulated in detail for the range 0 - 50 ^{o}C and 1 - 200 atmospheres corresponding properties of some other common liquids are shown in Table 2.1.

Units

The subject of this research is related to a number of scientific fields, namely Hydraulics, Thermodynamics and Physics.

In Hydraulics the concept of head or energy per unit weight is dominant. In Thermodynamics and Physics all properties are expressed in terms of unit mass. Physicists prefer the use of such units as slug, Newton et., which make 'g_u' (the constant of proportionality in Newtons Second Law of Motion) numerically equal to unity. On the other hand many mechanical and hydraulic engineers (especially those engaged in practical industrial work) prefer the use of units such as lb_m, lb_f, K_g, Kg_f so that numerical values of the weight and mass of a body are approximately equal on the surface of the earth.

To neutralise these differences the constant, g_u, is introduced in all equations where ambiguity may arise. A table of the values of g_u

is shown in the nomenclature. By introducing g_u the equations are made valid for any system of units; also the conversion from energy per unit mass to energy per unit weight or vice versa can easily be done by multiplying by (g_u/g) or (g/g_u) respectively.

Contributions which are believed to be of original nature

(1) A hydraulic bridge for measuring small temperature difference (Chapter 9); published:

Journal of Scientific instruments, Vol. 41 pp. 596-598 Oct. 1964

(2) The pair ratio method of measuring small temperature difference (Chapter 7); published:

Journal of Scientific Instruments, Vol. 41, pp. 592-595 Oct. 1967

(3) A general expression for the isentropic stagnation temperature of fluids (Chapter 3); published:

Nature, pp. 989-990, Vol. 204, No. 4962, 5th Dec. 1964

(4) A general expression for the overall recovery factor of streamline cylindrical pockets normal to the direction of flow (Chapter 10).

(5) Development of the heat diffusion methods of measuring the rate of mass flow (Chapter 12).

The following contributions are believed to be important improvements and additions to existing work:

(i) Presentation of the Steady Flow Energy Equation in a special form for convenient application to liquid flow (Chapter 4).

(ii) Calculation of the property $(\partial h/\partial P)_T$ of water for the temperatures 0 - 50°C and pressures 1 - 200 atmospheres (Tables 5.4 and 5.6)

(iii) Simple exposition of the principles of the thermodynamic methods of measuring the efficiency of hydraulic machines (Chapter 11).

(ii) and (iii) are published in

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Dr. A.S. Thom, Department of Aeronautics and Fluid Mechanics

Dr. T.R. Foord, Department of Electrical Engineering

Mr. R.C. Langlands, Department of Electrical Engineering

Mr. A.J. Binnie, Department of Electrical Engineering

and Mr. A.A.R. Elagib, Department of Aeronautics and Fluid Mechanics

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NOMENCLATURE

a, b	Resistance-temperature coefficients.
a_m	Coefficient of the monitor thermometer.
A_m	Equivalent coefficient of the monitor thermometer
a_s, a_T	The isentropic and the isothermal coefficients of pressure-enthalpy per unit mass.
A_s, A_T	The average isentropic and isothermal coefficients of pressure-enthalpy per unit mass.
A_s', A_T'	A_s/V_o , A_T/V_o
A_o	Equivalent area of heat transfer.
C	The hydraulic bridge constant.
c_p	The specific heat at constant pressure per unit mass.
C_p	The average specific heat at constant pressure per unit mass (mechanical units).
C_p'	C_p/J or C_p in heat units.
d	Diameter of the pocket.
D	Diameter of the pipe.
e	Total energy per unit mass.
f	Friction coefficient (Appendix 10.1)
g	Acceleration due to gravity.
g_o	Standard acceleration due to gravity (32.174 $ft^2/sec.$).

g_u	A constant dependent on the system of units used:-
	$= 32.174\ lb_m\ ft./lb_f\ s^2$ for lb_m/lb_f system
	$= 1.00\ lb_m ft./pdl.s^2$. for $lb_m/pdl.$ system
	$= 1.00$ slug $ft./lb_f s^2$ for slug/lb_f system
	$= 1.00\ g.cm./dyne\ s^2$ for g/dyne system
	$= 9.81\ Kg.m./Kg_f s^2$ for Kg/Kg_f system.
h	Enthalpy per unit mass.
h_c	Coefficient of heat transfer by natural convection.
H	$v_o \Delta P$.
H_f	Head losses due to friction (energy per unit weight).
J	Mechanical equivalent of heat ($J = 1400.2$ ft. lb_f/CHU).
k	Coefficient of thermal conductivity.
K_p	The pair ratio (Ch. 7).
K_t	The test ratio (Ch. 7).
K_m	The monitor ratio (Ch. 7)
l	Resistance of leads.
L	length
m	Coefficient of the hydraulic bridge.
n	Power index in the expression for the frictional recovery factor.
n_p, n_t	The pair ratio and the test ratio of the transformer ratio-bridge Appendix 10.1) $n = R_y/(R_y + R_x)$.

P	Absolute pressure.
P_a	Atmospheric pressure.
P_c	The average pressure coefficient.
$P_{c\theta}$	The pressure coefficient at θ^o to the direction of flow (clockwise) (Ch. 10).
P_∞	Pressure in the free stream.
P_r	The prandtl number ($\mu c_p/k$)
q	Heat transfer to the fluid per unit mass, (in mechanical units).
Q	Rate of flow per second (vol/sec).
Q_e	Heat transfer to the fluid per second (mechanical units).
Q_m	Rate of mass flow per second (mass/second).
r_o	Radius.
r_f	Frictional recovery factor.
$r_{f\theta}$	Local frictional recovery factor at θ^o (Ch. 10).
R	Resistance.
R_f	Overall recovery factor (Ch. 10).
R_n	Reynolds number (VD/ν).
R_o	Resistance of thermometer at 0^oC
R_T	Resistance of thermometer at T^oC
s	Entropy per unit mass.
S	Total entropy.
S_c	Coefficient of stagnation, $(T/v)(\partial v/\partial T)_p$ (Ch. 3).
t	Thickness.

T	Absolute temperature of the fluid.
$\overline{T}$	Temperature Celsius.
T_m	Measured temperature.
T_Θ	Local temperature on the surface of a non-conductive pocket at Θ^o to the direction of flow (Ch. 10)
T_∞	T, or temperature of the fluid at the free stream.
$T_{\infty\Theta}$	Temperature at the outer edge of the boundary layer of the pocket, at Θ^o to the direction of flow (Ch. 10).
T_R	Room temperature (Celsius).
u	Internal energy per unit mass.
U	Absolute velocity (in the direction of flow)
U_o	Absolute velocity, U, at Θ = o relative to pocket
U	Absolute velocity at the free stream
$U_{\infty\Theta}$	Velocity at the outer edge of the boundary layer and tangential to the pocket.
v	The specific volume (per unit mass)
v_o	The specific volume of water at 4^oC and atmospheric pressure v_o = 1 ml/gm = $1/62.43$ ft^3/lb_m
V	Average velocity Q/A
w	External work done by fluid per, unit mass
w_i	Ideal external work which can be done by the fluid per unit mass (frictionless flow).

w_f	Energy loss per unit mass due to degradation by friction ($w_f = (g/g_u)\, H_f$
W	External work done by fluid per second (mechanical units).
Z	Height above datum.
	Subscripts 1, 2 refer to the inlet and outlet of a flow system respectively.

Greek notations

α_o, δ	Callander resistance-temperature coefficient, (Ch. 7) .
θ	Angle measured clockwise to the direction of flow .
δ	error in .....
Δ	difference ....
ΔT	$T_2 - T_1$.
ΔT_R	$T_R - T$.
γ	The specific heat ratio C_p/C_v .
η, η_k	Efficiency, hydraulic efficiency (respect) .
η_{h_t}	Hydraulic efficiency of a turbine .
η_{h_p}	Hydraulic efficiency of a pump .
μ	Joule Thomson coefficient, $(\partial T/\partial P)_h$ (Ch. 2) .
ν	Kinematic viscosity (L^2/T) .

CHAPTER ONE

INTRODUCTION

1.1 The limitations of the available equations of Hydraulics

Hydraulics as known today is based mainly on the laws and equations of Mechanics, e.g., Newtons Second Law of Motion and its corollaries for fluid flow, Bernoulli and Eulers' equations. However it was always realised that these equations have serious limitations, being based on the assumption of inviscidity of the fluid and on the absence of external work. Thus in addition to these equations a form of 'total energy' or 'total head' equation is often quoted in text books of Hydraulics. This consists of Bernoulli or Euler's equation supplemented with the term for hydraulic losses. This term obviates only the assumption of inviscidity - but it does not of course account for the external work. When this has to be considered, for example, when measuring the efficiency of a hydraulic turbine, a long costly 'detour' has to be made instead of evaluating the external work directly from the change of the fluid properties. The detour involves the assessment of the characteristics of the generator and accurate measurement of the rate of discharge. To overcome these limitations a logical development might have been the employment of the more

general equations of thermodynamics. However, the use of such equations in practical Hydraulics would involve the measurement of temperature. Unlike the case of gas flow, the changes of temperature in liquid flow are very small and are difficult to measure accurately. Thus there was no alternative, but to exclude the use of these temperature-dependent thermodynamic equations.

1.2 The use of temperature as a variable in Hydraulics

From the preceding paragraph, it can be concluded that it is mainly a problem of measurement that inhibits the use of temperature as a variable in Hydraulics, and consequently inhibits the useful representation of the process of hydraulic flow by the more general equations of thermodynamics. The first objective of the research reported in this thesis was to develop simple methods for the accurate measurement of small temperature differences such as those occurring in a hydraulic flow system.

The second objective was to apply the basic principles of thermodynamics to hydraulic flow. This

implies the adoption of the general thermodynamic steady flow energy equation instead of the idealised equations of Fluid Mechanics. As will be shown later, this equation has to be recast in a special form for convenient application to hydraulic flow.

The third objective was to show the practicability and the advantages of using this general thermodynamic equation and of using the temperature as a variable in two important fields of hydraulic practice, namely, the measurement of efficiency and of the rate of flow.

The following thesis is thus divided into three parts in accordance with the above objectives.

PART A

THEORY

APPLICATION OF THE BASIC PRINCIPLES OF THERMODYNAMICS TO HYDRAULICS

CHAPTER TWO

SOME DISTINCTIVE FEATURES OF LIQUID THERMODYNAMICS

2.1 Incompressibility

A study of liquids would hardly be complete without an introductory discussion of this distinctive liquid behaviour, to which the science of Hydraulics owes its historic existence long before many other sciences.

In mathematical terms incompressibility may be defined as

$$v = \text{a constant, or } dv = 0.$$

Surprisingly, for liquids these simple equations are as important as the perfect gas law is for gases. Both are approximate, but are justifiable and useful assumptions.

No liquid is actually incompressible, but the deviation from this condition is very small as can be seen from applying the following equation.

For a mono-phase substance, $v = f(P,T)$

and for any thermodynamic process,

$$(dv/v) = (1/v)\left[(\partial v/\partial P)_T dp + (\partial v/\partial T)_p dT\right]$$

$-(1/v)(\partial v/\partial P)_T$ and $(1/v)(\partial v/\partial T)_p$ are the isothermal

compressibility and the isobaric (constant pressure) coefficient of thermal expansion respectively. For water at 15°C and atmospheric pressure these are equal to about 4.7×10^{-5} per atmosphere and 1.5×10^{-4} per degree °C respectively. For air, if the perfect gas law is assumed, the coefficients are equal to (1/P) and (1/T). At 15°C and atmospheric pressure these are of the order of 1.0 per atmosphere and 3.5×10^{-3} per degree °C.

The above example clearly justifies the assumption of incompressibility of liquids in Hydraulics, where only the variation of density or specific volume is in question. Thus the assumption is effectively used when integrating 'νdp' or (dp/ρ) to obtain the head in Bernoulli equation or when comparing the velocities in two sections of a 'stream-tube' of different areas. In this latter case it is safe to assume that at constant rate of mass flow the velocity or kinetic energy is only a function of the area (VA = a constant), and that it is independent of any thermodynamic or heat transfer process taking place between the two sections. The thermodynamic significance of this lies in the fact that the inlet and outlet conditions in a reversible process (except

the temperature) are assumed to be typical to those in an irreversible process. Thus the available head for a turbine, which may be defined as the output of an ideal frictionless machine, can be assessed from the measurements made on the actual machine.

For a thermodynamic treatment of liquid flow considering variations in temperature, the assumption of incompressibility has to be carefully reviewed. It must not imply, for example, that the temperature change, due to isentropic expansion or compression, is negligible. This temperature change, although small, may be of the same order as that in the viscous process being studied. For example, in an efficiency test on a turbine supplied with water at $20^{\circ}C$, such assumption of incompressibility may cause an error of not less than 6%.

2.2 The Change of Enthalpy

From the two-property rule, the thermodynamic state of a pure substance and hence all its properties may be fixed if two of them are known. Thus the enthalpy can be defined as a function of any two of the other properties. Pressure and temperature will

be chosen because they are the least difficult to measure when the fluid is in motion.

For a mono-phase substance $h = f(P,T)$

and $$dh = \left(\frac{\partial h}{\partial P}\right)_T dp + \left(\frac{\partial h}{\partial T}\right)_p dT \qquad (2.2)$$

For the purpose of the following analysis $(\partial h/\partial P)_T$, the isothermal coefficient of pressure enthalpy, will be denoted by a_T. $(\partial h/\partial T)_P$, the specific heat will be denoted as usual by c_p.

From appendix (2.1) $$a_T = (\partial h/\partial P)_T = \left[v - T(\partial v/\partial T)_P\right] \qquad (2.3)$$

Thus equation (2.2) may be recast in its more usual form:-

$$dh = \left[v - T(\partial v/\partial T)_\rho\right] dP + c_p dT \qquad (2.4)$$

As will be shown later this equation provides one of the most useful arguments in liquid thermodynamics or "thermo-hydraulics."

It is worth noting that for a gas obeying the perfect gas law ($Pv = \bar{R}T$), the term $T(\partial v/\partial T)_p$ is equal to v, thus $(\partial h/\partial P)_T$ or $\left[v - T(\partial v/\partial T)_P\right]$ is equal to zero. Hence

$$dh = c_p dT$$

This is the equation often used for the change of enthalpy of air and most other gases. On the other

hand, because $(\partial v / \partial T)_p$ for a liquid is usually very small, $T\ (\partial v / \partial T)_p$ for most liquids at normal room temperatures is only a fraction of v (Table 2.1). Consequently the term $\left[v - T\ (\partial v / \partial T)_p \right] dP$ constitutes an important portion of the change of enthalpy. This is a distinctive contrast between gas and liquid thermodynamics.

2.3 The constant-enthalpy Temperature Pressure relation. (The Joule-Thomson Coefficient, $(\partial T / \partial P)_h$)

From equation (2.4),

$$(\partial T/\partial P)_h = \left[T(\partial v / \partial T) - v \right]/c_p = - (a_T/c_p) \qquad (2.6)$$

$$= \mu_h$$

The Joule-Thomson coefficient is an important thermodynamic identity which has been studied in detail for many gases and vapours. It has many practical applications, e.g. in gas liquefaction by throttling and in the determination of the specific heat (Ref. 1). The latter application is also possible in case of liquids as described in Chapter (5).

An interesting contrast between liquid and gas behaviour is displayed by equation (2.6). From the structure of the equation it can be seen that for a perfect gas the Joule-Thomson coefficient is zero. It can also be inferred that for gases not obeying the perfect gas law it is likely to be positive while for liquids it tends to be negative. This implies that a gas cools while a liquid heats up on throttling. Indeed this is the case for most common fluids at room temperatures, although at high temperatures and/or high pressures some fluids tend to have a point of inversion where the sign of the coefficient changes.

The relation for $(\partial T/\partial P)_h$ provides the theoretical basis of two novel devices developed in this research, namely the hydraulic throttling calorimeter and the hydraulic bridge described in Chapters (8) and (9) respectively. The former is used for standardisation and calibration of temperature measuring apparatus. The latter is used for measurement of small temperature difference. The theory of both devices is based on the fact that the Joule-Thomson coefficient represents the temperature/pressure ratio in the constant enthalpy process of throttling. Thus when its value is known, the small temperature change can be assessed precisely from the

corresponding simple pressure measurements.

The value of the coefficient for water may be obtained by the aid of Tables (5.4) and (5.7). At about 15°C and atmospheric pressure the value of the coefficient is about 1.6 m°C per lb/in^2(millidegree, m°C = 0.001°C).

2.4 The constant-entropy Temperature/Pressure relation, $(\partial T/\partial P)_s$

From Appendix (2.2),

$$(\partial T/\partial P)_s = (T/c_p)(\partial v/\partial T)_p \qquad (2.7)$$

Like the Joule-Thomson coefficient, this is a useful thermodynamic quantity. It provides the means for estimating the reversible part of the temperature change in any process. It also provides the means for assessment of the temperature change due to an isentropic acceleration or deceleration of flow. This application will be discussed in detail in the following Chapter and later in the text.

The coefficient $(\partial T/\partial P)_s$ is positive for both gases and liquids, except for water below 4°C and at low pressures (Refer to Tables (5.3) and (5.7).) The value of the coefficient for liquids is much smaller as may be judged from the equation.

For a perfect gas the above relation is usually represented by the equation

$$(T_2/T_1) = (P_2/P_1)^{\frac{\gamma-1}{\gamma}} \qquad (2.8)$$

An interesting application of equation (2.7) is in the estimation of the temperature gradient in an adiabatic quiescent mass of fluid, i.e. a fluid with no (or negligible) mass transfer or heat interchange between its layers. In this case the change of temperature with depth is assumed to be analogous to that in an isentropic compression of the fluid.

By the aid of the relation,

$$v\,dP = - (g/g_u)\,dZ, \qquad (2.9)$$

equation 2.7 may be recast as,

$$(dT/dZ) = -(T/v)(\partial v/\partial T)_p (1/c_p)(g/g_u).$$

For air in a quiescent atmosphere at 15°C and standard g,

$$(dT/dZ) = -(1/c_p)(g/g_u) \qquad (2.11)$$

(assuming $Pv = \hat{R}T$)

$\doteq$ 1°C rise per 336 ft. decrease of altitude.

For water in a quiescent ocean at the same conditions of air above,

(dT/dZ) = 0.048 °C rise per 1000 ft. increase of depth. (2.12)

(This figure applies to salty water containing 3.5 %. salt.)

APPENDIX (2.1)

VALUES OF THE COEFFICIENTS OF PRESSURE-ENTHALPY

The Value of a_T

By definition $h = u + Pv$; (A.I.1)

differentiating, $dh = du + Pdv + vdp$ (A.I.2)

From the First Law of Thermodynamics and the definition of entropy:

$$Tds = du + Pdv \quad \text{(A.I.3)}$$

Thus equation (A.I.2) becomes

$$dh = Tds + vdp \ , \quad \text{(A.I.4)}$$

Introducing the Gibbs Function:

$$G = h - Ts \ ; \quad \text{(A.I.5)}$$

and differentiating gives $d G = dh - Tds - sdt$. (A.I.6)

From (A.I.4)

$$dh - Tds = vdp$$

hence equation (A.I.6) becomes:

$$dG = vdp - sdT. \quad \text{(A.I.7)}$$

From the rules of Partial Differentiation

if $G = f\ (P \ \& \ T)$,

$$\left(\frac{\partial \left(\frac{\partial G}{\partial P} \right)_T}{\partial T} \right)_P = \left(\frac{\partial \left(\frac{\partial G}{\partial T} \right)_P}{\partial P} \right)_T \quad \text{(A.I.8)}$$

Applying equation (A.I.8) to (A.I.7),

$$-\left(\frac{\partial v}{\partial T}\right)_P = \left(\frac{\partial s}{\partial P}\right)_T \qquad \text{(A.I.9)}$$

(one of Maxwell's relations ref. 2)

From equation (A.I.4) and the definition of a_T,

$$a_T = \left(\frac{\partial h}{\partial P}\right)_T = v + T\left(\frac{\partial s}{\partial P}\right)_T ; \qquad \text{(A.I.10)}$$

substituting for $\left(\frac{\partial s}{\partial P}\right)_T$ from equation (A.1.9),

$$a_T = \left(\frac{\partial h}{\partial P}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_P ; \qquad \text{(A.I.10)}$$

If for a gas the law $Pv = \bar{R}T$ is assumed, then $T\left(\frac{\partial v}{\partial T}\right)_P$ will be equal to v, hence a_T or $\left(\frac{\partial h}{\partial P}\right)_T$ will be zero.

Equation (2.2) becomes;

$$dh = \left(\frac{\partial h}{\partial T}\right)_P = c_P\, dT \qquad \text{(A.I.12)}$$

The Value of a_s

From equation (A.I.4)

$$\left(\frac{\partial h}{\partial P}\right)_s = v \qquad \text{(A.I.13)}$$

By definition $a_s = \left(\frac{\partial h}{\partial P}\right)_s$ hence,

$$a_s = v \qquad \text{(A.I.14)}$$

APPENDIX (2.2)

Value of $(\partial T/\partial P)_s$

By the two-property rule

$$s = f(P,T) \qquad (A.2.1)$$

$$ds = (\partial s/\partial P)_T \, dp + (\partial s/\partial T)_P \, dT \qquad (A.2.2)$$

$$\therefore (\partial T/\partial P)_s = - (\partial s/\partial P)_T \, / \, (\partial s/\partial T)_P \qquad (A.2.3)$$

By substituting for $-(\partial s/\partial P)_T$ by $(\partial v/\partial T)_P$ from equation (A.1.9) in Appendix 2.1 above equation (A.2.3) is recast as

$$(\partial T/\partial P)_s = (\partial v/\partial T)_P / (\partial s/\partial T)_P \qquad (A.2.4)$$

Multiply numerator and denominator of right hand side of equation (A.2.4) by T, and substitute c_P for $T(\partial s/\partial T)_P$

$$(\partial T/\partial P)_s = (T/c_P)(\partial v/\partial T)_P \qquad (A.2.5)$$

TABLE 2.1 Thermodynamic properties of some common liquids at atmospheric pressure and 15 °C.

	v/v_0	$\frac{1}{v}(\partial v/\partial T)_p$ $\times 10^4$ Per °C	$-\frac{1}{v}(\partial v/\partial P)_T$ Per atm.	c_p ft.lb$_f$/C°	$\frac{1}{v}(\partial h/\partial P)_T$ or a_T/v	$(\partial T/\partial P)_h$ $-a_T/c_p$ or 10^{-3} °C/lb.in^{-2}	$(\partial T/\partial P)_s$ C° / lb.in^{-2}
Water	1.001	1.5	4.7×10^{-5}	1400	0.958	- 1.58	7.2×10^{-5}
Sea water (3.5% Salt)	0.975	2.2		1305	0.940	- 1.50	11.2×10^{-5}
No. 1 Petrol ν = 0.71 Centistokes	1.35	11.2		660	0.675	- 3.2	153×10^{-5}
Diesel (or gas) oil ν = 6.6 Centistokes	1.15	7.6		680	0.782	- 3.1	86×10^{-5}
Perfect gas	(Air) 825	35	1.00	(Air) 336	0	0	(Air) 5.65

$v_0 = 1$ ml/gm $= 1/62.43$ ft^3/lb$_m$

CHAPTER THREE

A THERMODYNAMIC TREATMENT OF LIQUID FLOW

3.1 Euler, Bernoulli and the steady flow energy equation

In Chapter (1) it was mentioned that both Euler and Bernoulli's equations had been developed from the laws of mechanics only. Thus to derive them the assumption of inviscidity of the fluid and the absence of external work had to be made. A more general equation suitable for fluid flow is the steady flow energy equation (denoted sometimes by S.F.E.E.) This equation is developed from the law of conservation of energy or the First Law of Thermodynamics. For a pure substance in the absence of capillarity, electricity and magnetism this equation may be written in the form

$$dw = dq - \underbrace{\left[dh + (1/g_u)\, VdV + (g/g_u)\, dZ\right]}_{\text{energy per unit mass}} \quad (3.1.a)$$

$$\text{or } (g_u/g)\, dw = (g_u/g)\, dq - \underbrace{\left[(g_u/g)\, dh + (1/g)VdV + dZ\right]}_{\text{energy per unit weight}} \quad (3.1.b)$$

Euler and Bernoulli's equations may be derived from the steady flow energy equation in the following manner.

The S.F.E.E. is written for inviscid flow:

$$dw_i = dq - \left[dh + (1/g_u)\, VdV + (g/g_u)\, dZ\right] \tag{3.2}$$

The subscript i denotes that the corresponding change is taking place in an ideal frictionless flow.

From the First Law of Thermodynamics and the definition of entropy and enthalpy,

$$Tds = dh - vdp.$$

For frictionless flow the change of entropy is due only to heat transfer. Hence

$$dq = Tds = dh - vdp \tag{3.3}$$

The substitution of this value of dq in equation (3.2) gives

$$dw_i = -\left[vdp + (1/g_u)\, VdV + (g/g_u)\, dZ\right] \tag{3.4}$$

dw_i is the maximum or ideal work obtainable from the flow. In the absence of this external work equation (3.4) gives Euler's equation

$$vdp + (1/g_u)\, VdV + (g/g_u)\, dZ = 0 \tag{3.5}$$

It is worth noting that the validity of equation (3.5) (or Euler's equation) is not affected by the presence or absence of heat transfer. By the integration

of Euler's equation and with the assumption that v or $1/\rho$ = a constant, the well known form of Bernoulli's equation is obtained.

$$P_1/\rho + V_1^2/2g_u + (g/g_u)\, Z_1 = P_2/\rho + V_2^2/2g_u + (g/g_u)\, Z_2 \qquad (3.6)$$

3.2 A thermodynamic expression for the hydraulic losses

To overcome the limitations imposed by the assumption of inviscidity, the following 'total energy' version of Bernoulli's equation is often used in text books of Hydraulics for flow in the absence of external work:-

$$P_1/\rho + V_1^2/2g_u + (g/g_u)\, Z_1 = P_2/\rho + V_2^2/2g_u + (g/g_u)\, Z_2 + (g/g_u)\, H_f \qquad (3.7)$$

H_f in equation (3.7) is the hydraulic or head losses (energy per unit weight) due to friction and shear forces between the points 1 and 2. It represents the portion of energy which is degraded or turned into the lower grade thermal energy. Also it can be considered

as a measure of the irreversibility of the flow in the sense that it cannot be converted back into any of the three forms of energy included in the equation.

In the absence of external work, H_f may be assessed directly by using equation (3.7). But a useful expression for general application may be obtained by using the foregoing more general thermodynamic equations, (3.1.a) and (3.4).

From the preceding thermodynamic explanation of H_f it may be defined as the difference between the maximum work which is obtainable from the flow - namely the ideal work in the frictionless flow - and the work which is actually produced by the flow:-

$$(g/g_u)\, d H_f = d w_f = d w_i - dw \qquad (3.8)$$

From equations (3.1.a) and (3.4),

$$d w_f = dh - vdp - dq \qquad (3.9)$$

From equation (2.4) for dh and equation (3.9)

$$d w_f = \left[c_p dT - T(\partial v / \partial T)_P dP \right] - dq \qquad (3.10)$$

To obtain equations (3.9) and (3.10) it is assumed that the kinetic energy-terms at the end of a reversible and an irreversible process are the same (Section 2.1).

Due to the short time in which the process of heat transfer takes place, and due to the low potential difference between the room and the fluid temperature in most of the fields of hydraulic practice considered in this thesis, dq may be neglected (Refer to Section 6.7) Thus equation (3.10) may be re-written as

$$d w_f = \left[c_p \, dt - T \, (\partial v / \partial T)_p dP \right] \quad (dq = o) \tag{3.11}$$

A very instructive form of equation (3.11) may be obtained by substituting for $T \, (\partial v / \partial T) \, dp$ from the thermodynamic relation,

$$(\partial T / \partial P)_s = (T/c_p)(\partial v / \partial T)_p \tag{2.7}$$

with the result that

$$d w_f = c_p \, (d T - dT_s) \tag{3.12}$$

$(dT - dT_s)$ is the difference between the change of temperature in the actual viscous process and that resulting from the isentropic expansion or compression of the fluid.

It is worth noting that equations (3.11) and (3.12) are applicable irrespective of the presence or absence of external work.

3.3 A general expression for the isentropic stagnation temperature of a fluid *

The steady flow energy equation applied to an isentropic flow gives:-

the isentropic change of enthalpy $(dh)_s = v(dp)_s$

$$= - UdU \quad (3.13)$$

From Maxwell's thermodynamic relations (Ref. 2) and the definition of specific heat at constant pressure (or from Appendix 2),

$$(\partial T/\partial P)_s = (T/c_p)(\partial v/\partial T)_P \quad (2.7)$$

By substituting for $(dp)_s$ from equation (3.13) and rearranging, equation (2.7) becomes

$$(dT)_s = - (T/v)(\partial v/\partial T)(UdU/c_p) \quad (3.14)$$

If $(T/v)(\partial v/\partial T)_p(1/c_p)$ is assumed a constant (or if an average value is adopted for it), the integration of equation (3.14) between the free-stream and stagnation conditions gives:-

$$T_o = T_\infty + (T/v)(\partial v/\partial T)_p(U_\infty^2/2c_p) \quad (3.15)$$

For afluid obeying the perfect gas law the stagnation coefficient $(T/v)(\partial v/\partial T)_p$ is equal to

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* Elagib, The Journal 'Nature,' (Ref. 3)

unity; only then is the following equation, which is often quoted in text books for the stagnation temperature, is applicable.

$$T_0 = T_\infty + U_\infty^2 / 2c_p \qquad (3.16)$$

For water at atmospheric pressure, the coefficient varies from about 0.04 at 14°C to about 0.12 at 50°C; (Refer to Table (5.1) for values of v and to Table (5.3) for values of $T\,(\partial v / \partial T)_p$,) but at high pressures and temperatures it may exceed that of a perfect gas. For, example at 200 atmospheres and 300°C, the coefficient reaches 1.43. An even more interesting fact is that at temperatures between 0°C and 4°C and low pressures, the stagnation coefficient for water is negative, (because $(\partial v / \partial T)_P$ is negative). Thus, contrary to expectation, the stagnation temperature is less than that of the free stream.

This discussion of the stagnation temperature will be referred to later, in Chapter 10 when the 'thermometer problem' is discussed.

3.4 A thermodynamic illustration of a hydraulic flow system

As a summary of the above thermodynamic treatment of hydraulic flow, a thermodynamic illustration of a hydraulic flow system is presented in Fig. 3.1. The diagram represents a hydro-electric power scheme. The energy balance is represented by the steady flow energy equation outlayed in the form

$$-dZ + (g_u/g)\,dq = (g_u/g)dw + (g_u/g)\,v\,dp + (1/g)\,VdV + (g_u/g)\left[C_p dT - T(\partial v/\partial T)_p\,dP\right]$$

(energy per unit weight) (3.17)

The terms are presented in units of energy per unit weight so that they appear in units of length or height in the normal manner followed in hydraulic practice. The equation is formulated in such form from the relation $dw_i = dw + dw_f$ of equation (3.8) and from equations (3.4) and (3.10) for dw_i and dw_f respectively.

If the heat transfer is neglected equation (3.17) may be reduced to

$$-dZ = (g_u/g)\,dw + (g_u/g)\,vdp + (1/g)\,VdV + (g_u/g)\left[c_p dT - T\,(\partial v/\partial T)_p dp\right]$$

(3.18)

To assess the maximum work obtainable from the scheme equation (3.4) is integrated between the points 5 and 1, giving

$$\int_1^5 (g_u/g)\, d w_i = -\int_1^5 dZ - \int_1^5 (g_u/g)\, v\, dp$$

$$(V_1 = V_5 = 0) \qquad (3.19)$$

If 'g' and the atmospheric pressure are assumed constant between the reservoir and the tailrace levels, equation (3.19) may be reduced to

$$(g_u/g)\ w_i = (Z_1 - Z_5) \qquad (3.20)$$

enenrgy per unit weight of fluid.

It is worth noting that the presence or absence of heat transfer does not affect the validity of equation (3.19) or (3.20).

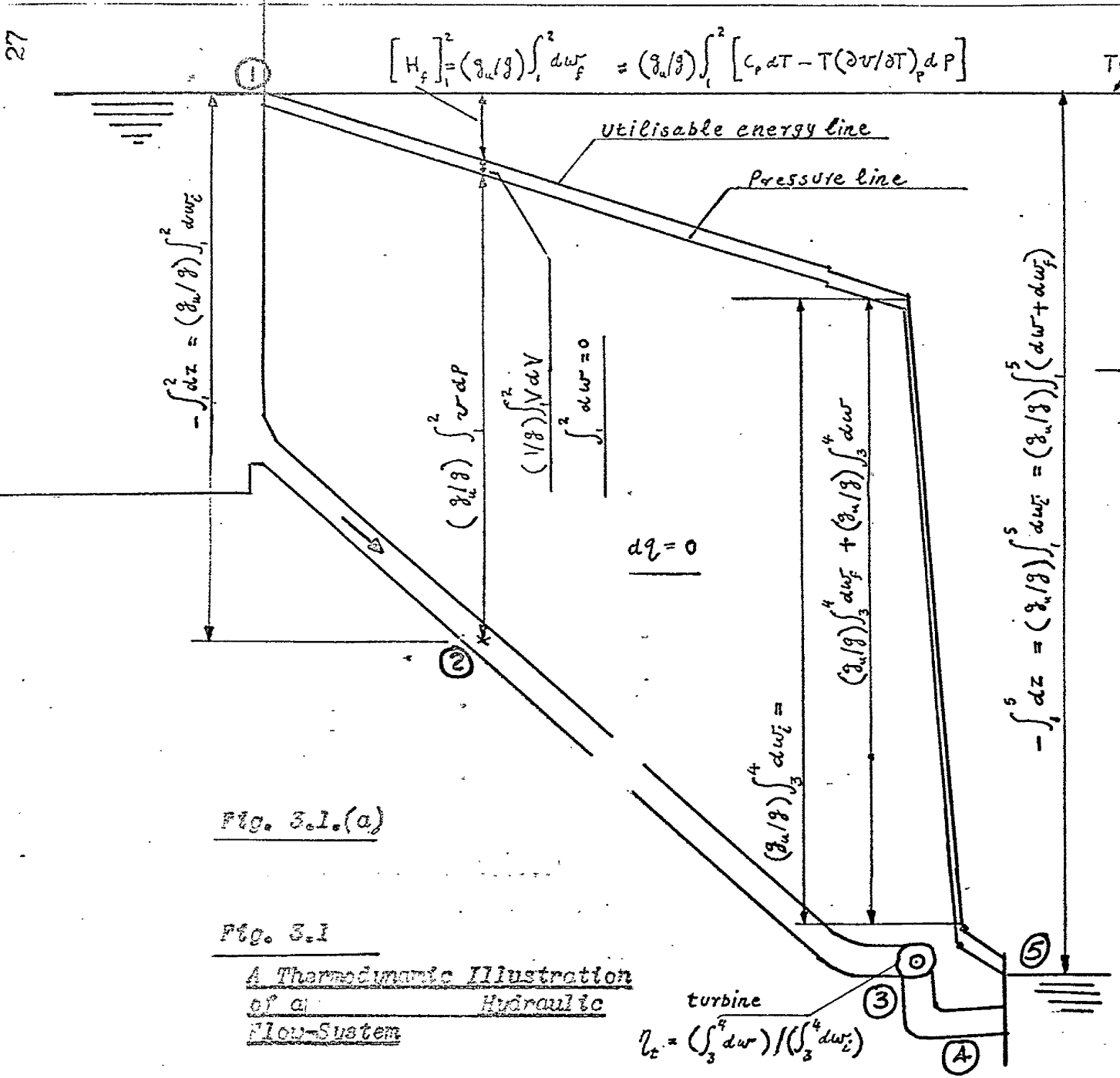

Fig. 3.1.(a)

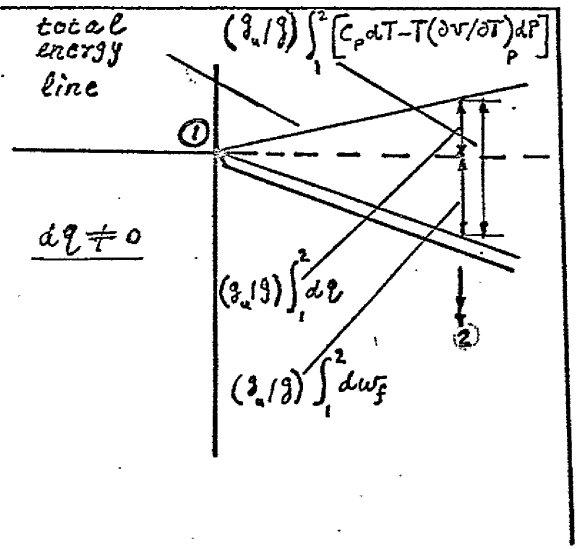

Fig. 3.1.(b)

Effect of Heat Transfer

Fig. 3.1

A Thermodynamic Illustration of a Hydraulic Flow-System

CHAPTER FOUR

A SPECIAL FORM OF THE STEADY FLOW ENERGY EQUATION FOR APPLICATION TO HYDRAULIC FLOW

Consider the hydraulic flow system illustrated in Fig. (4.1), which may represent a machine producing or absorbing external work, a throttling valve, an orifice or may just represent a length of piping. The S.F.E.E. (3.1.a) when integrated between points (1) and (2) gives:-

$$w = (h_1-h_2)+(1/2g_u)(V_1^2-V_2^2)+(g/g_u)(Z_1-Z_2) + q \qquad (4.1)$$

As explained in section (3.2) the heat transfer q may be neglected. Both the kinetic and position energy terms in the right hand side of equation (4.1) are measurable by normal mechanical means. To assess the value of the enthalpy term the usual methods followed for gases, e.g. the direct use of enthalpy-entropy charts, cannot be applied conveniently because the pressure and/or the temperature differences involved in hydraulic flow are relatively very small. To make use of such charts, they need to be very large and very detailed. However, reports by Rögner* showed that satisfactory results are obtainable if these

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* (Ref. 4)

methods are applied to feed water pumps. Here, again the pressure and/or the temperature differences are high and are of the same order as those occurring in gas flow. Thus for normal hydraulic practice a special technique has to be devised. This is achieved by breaking up the enthalpy change and expressing it as in equation (2.2):

$$dh = (\partial h/\partial P)_T dp + (\partial h/\partial T)_p dT \qquad (2.2)$$

$$= a_T dp + c_p dT \qquad \text{(Section 2.2)}$$

Integrating,

$$(h_1-h_2) = A_{T_1}(P_1-P_2) + C_{P_2}(T_1-T_2), \qquad (4.2)$$

where

$$A_{T_1} = \left(1/(P_1-P_2)\right) \int_{P_2}^{P_1} a_{T_1}\, dp,$$

the average isothermal coefficient of pressure enthalpy

and

$$C_{P_2} = \left(1/(T_1-T_2)\right) \int_{T_2}^{T_1} c_{p_2}\, dT,$$ the average specific heat. (4.3)

Thus equation (4.1) may be re-written as

$$w = A_{T_1}(P_1-P_2) + C_{p_2}(T_1-T_2) + (1/2g_u)(V_1^2-V_2^2) + (g/g_u)(Z_1-Z_2) \qquad (4.4)$$

$(q = o)$

The arrangement of the S.F.E.E. in this manner reduces the need for accurate measurement of absolute temperature to that of the temperature difference. The absolute temperature needs only be measured approximately.

If the flow is isentropic, the integration of the S.F.E.E. gives

$$w_i = (h_1-h_2)_s+(1/2g_u)(V_1^2-V_2^2)+(g/g_u)(Z_1-Z_2) \qquad (4.5)$$

$$(h_1-h_2)_s = \int_2^1 (\partial h/\partial P)_s \, dp = \int_2^1 v\,dp \qquad (4.6)$$

Appendix (2.1)

The subscript 's' denotes that the flow is isentropic (or assumed to be isentropic) w_i is the reversible or ideal work (Section 3.1). In consistence with the preceding notations for the partial derivatives of enthalpy,

$$(h_1-h_2)_s = A_s (P_1 - P_2) \qquad (4.7)$$

where

$A_s = (1/(P_1-P_2)) \int_2^1 a_s dp$, the average isentropic coefficient of pressure enthalpy. (4.8)

Thus equation (4.5) may be re-written as

$$w_i = A_s(P_1-P_2)+(1/2g_u)(V_1^2-V_2^2)+(g/g_u)(Z_1-Z_2) \qquad (4.9)$$

The properties, a_T, A_T, a_s, A_s, c_p and C_p for water are presented in Tables (5.1 - 5.7). (Refer to Chapter 5).

4.2 Application of the Steady Flow Energy Equation

The special version of the S.F.E.E. formulated above may be used to evolve the thermodynamic expression for the hydraulic efficiency:-

Turbine

The hydraulic efficiency, η_t =

The external work produced by the actual machine
The external work producible by an ideal frictionless machine

$$= (w) / (w_i) \qquad (4.10)$$

From equations (4.4), (4.9) and (4.10)

$$\eta_t = \frac{A_{T_1}(P_1-P_2)+C_{p_2}(T_1-T_2)+(1/2g_u)(V_1^2-V_2^2)+(g/g_u)(Z_1-Z_2)}{A_s(P_1-P_2) + (1/2g_u)\ (V_1^2-V_2^2) + (g/g_u)\ \ (Z_1 - Z_2)}$$

(4.11)

Pump

The hydraulic efficiency η_p, =

$$= \frac{\text{the external work required by an ideal frictionless machine}}{\text{the external work required by the actual machine}}$$

$$= (-w_t) / (-w) \qquad (4.12)$$

$$= \frac{A_s(P_2-P_1)+(1/2g_u)(V_2^2-V_1^2) + (g/g_u)(Z_1 - Z_2)}{A_{T_1}(P_2-P_1)+C_{p_2}(T_2-T_1)+(1/2g_u)(V_2^2-V_1^2)+(g/g_u)(Z_2-Z_1)} \qquad (4.13)$$

Equations (4.11) and (4.13) form the basis of the thermodynamic methods of measuring efficiency described in Chapter 11. The analysis leading to these equations is simpler and much more direct than the available exposition by Willm and Campmas (Ref. 5 or as quoted in Ref. 24).

Throttle-valve

For a throttle valve, an orifice, a constriction or a section of piping, w = o. Hence equation (4.4) reduces to

$$A_{T_1}(P_1-P_2)+ C_{p_2}(T_1-T_2)+(1/2\ g_u)(V_1^2-V_2^2)+(g/g_u)(Z_1-Z_2) = 0 \qquad (4.14)$$

This equation may also be obtained from equation (4.11) by considering the valve or constriction as a turbine of zero efficiency.

Equation (4.14) forms the basis of the hydraulic throttling calorimeter and the hydraulic bridge described in Chapters (8) and (9) respectively

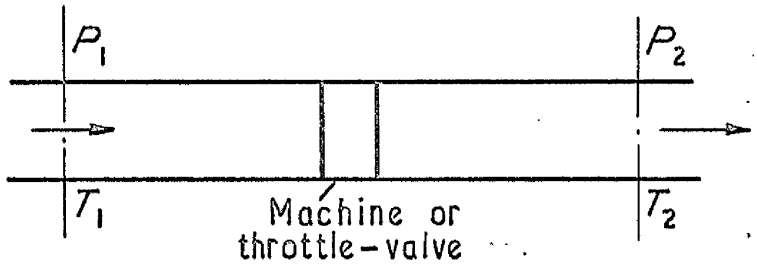
Hydraulic losses

The hydraulic losses as defined in Section (3.2) may be determined for any section of the flow system by using equations (4.4) and (4.9).

$$w_f = w_i - w = C_{p_2}(T_2-T_1) + (A_s-A_{T_1})(P_1 - P_2) \quad (4.15)$$
$$(q = o)$$

Like equation (3.11) equation (4.15) is valid irrespective of the existence or absence of external work.

FIG. 4.1. A FLOW SYSTEM



CHAPTER FIVE

THE AVERAGE COEFFICIENTS OF ENTHALPY,

5.1 Evaluation and tabulation of the coefficients for water

The average coefficients A_s , A_T and C_p for water are calculated and presented in Tables (5.1 - 5.7) in the form A'_s , A'_T and C'_p respectively, where

$$A'_s = A_s / v_o \quad , \quad A'_T = A_T / v_o \quad \text{and} \quad C'_p = C_p / J$$

$$v_o = 1 \text{ ml/gm} = (1/62.43) \text{ ft}^3/\text{lb} \qquad (5.1)$$

$$J = 426.6 \text{ Kg}_f \text{ m/Kcal} = 1400.2 \text{ ft. lb}_f/\text{CHU}$$

This precedure makes the tabulation independent of the system of units. The choice of v_o makes the values of A_s and A_T consistent with the thermodynamic correction factors suggested by Willm and Campmas (Ref. 5) in their analysis of the thermodynamic methods of measuring efficiency. (Refer also to Chapter 11).

Values of v , a_T and $T(\frac{\partial v}{\partial T})_P$ are presented in Tables (5.1 - 5.4), also as ratios of v_o . These values apply to pure water or ordinary water which may be considered as reasonably pure. If however, extreme

contamination with impurities or dissolved air is suspected, tests have to be carried out to correct the tabulated values.

For the construction of the tables, all the values of v and $(\partial v / \partial T)_P$ are taken or interpolated from Amagat's tables,* except the values of $(\partial v / \partial T)_P$ at atmospheric pressure between 0 - 5°C. These are taken from the revised table of Chappuis* which is more detailed than Amagat's. Values of c_p at atmospheric pressure and higher pressures are obtained from Tables compiled by Dorsey* after Osborne and after Koch respectively. Intermediate values are obtained by parabolic interpolation, where this is not applicable (e.g. near 4°C) the values are obtained by drawing fair curves.

The integrations $\int_{P_2}^{P_1} (\partial h/\partial P)_s \, dp$, $\int_{P_2}^{P_1} (\partial h/\partial P)_T \, dp$ are worked out by Simpson's Rule, with the assumption that the lower pressure is atmospheric. In cases where this is not so, e.g. in the case of throttle-valves and multi-stage pumps, then

$$\int_2^1 (\partial h/\partial P)_T \, dp = (A_T)_{P_1}(P_1 - P_a) - (A_T)_{P_2}(P_2 - P_a), \qquad (5.2)$$

* These tables are compiled by Dorsey Ref. 6.

where P2, is the lower pressure (which is not atmospheric), $(A_T)_{P_1}$ and $(A_T)_{P_2}$ are the values of A_T for the denoted pressure P_1 and P_2 respectively, and P_a is the standard atmospheric pressure (14.696 16 /in^2 or 1.0332 Kg /cm^2). The same procedure is applicable in case of A_s.

In reading the value of C_p the average value of T_1 and T_2 is to be used. In case of A_s' and A_T' the initial temperature may be used. The error so involved in the value of A_s' is negligible (maximum error is of the order of 1×10^{-4}% per atmosphere).

An example of the method of calculation of these properties is shown in appendix (5.1).

5.2 Proposed Methods for the experimental determination of the average coefficients of enthalpy of liquids, A_T & C_p.

(a) The constant enthalpy experiment

A hydraulic throttling calorimeter such as that described in Chapter (8), may be used here. It consists of a thermally insulated length of a horizontal uniform pipe, with a valve or orifice at its middle. A diagramatic illustration of the overall hydraulic circuit is shown in Fig. (5.1). The circuit comprises a high head pump, a thermostatic tank and a

weight ram for adjustment of the circuit absolute temperature and pressure respectively.

The liquid is throttled and the pressure and temperature differences across the throttle are measured, the latter being measured by any of the methods described in Part B of this thesis. The average Joule-Thomson coefficient, μ_h is determined from the equation

$$(T_2-T_1) / (P_1 - P_2) = A_T / C_p = -\mu_h \quad (5.3)$$

This equation is obtained by the aid of equations (2.6) and (4.3). This experiment gives values of μ_h for the ranges of pressure and temperature needed. As μ is only a ratio of the two properties A_T and C_p another experiment is needed before either property can be determined.

(b) <u>The Constant change of enthalpy experiment</u>

An electric immersion heater may be installed inside the calorimeter pipe instead of the throttle-valve. From equation (4.1),

$$q = h_2 - h_1, \quad (w = o, \; Z_1 = Z_2 \text{ and } V_1 = V_2) \quad (5.4)$$

$(h_2 - h_1)$ is equal to the electric input to the heater per unit mass. If means are provided for the measurement of this input and the rate of flow, $(h_2 - h_1)$ can be determined.

By the two property-rule,

$$T = f(P, h)$$

$$dT = (\partial T/\partial P)_h dP + (\partial T/\partial h)_P dh$$

The integration of this equation gives

$$T_2 - T_1 = \mu_h (P_2 - P_1) + (1/C_p)(h_2 - h_1) \qquad (5.5)$$

Hence

$$C_p = (h_2 - h_1) / \left[(T_2 - T_1) + \mu_h (P_1 - P_2)\right] \qquad (5.6)$$

The temperature and pressure differences are measured as before. The appropriate value of μ_h is selected from those determined by experiment (a) above. Equation (5.6) then gives the value of the average C_p. Subsequently A_T may be determined from the relation $A_T = -\mu_h \times C_p$

The above methods of determining the coefficients of enthalpy are analogous to those used in the Mechanical Engineering Department, University of Glasgow, for the determination of the properties of steam (Ref. 1); but as mentioned in section

(2.3) steam, cools on throttling. Thus by installing a heater downstream of the throttle-valve, and by adjusting the output of this heat T_2 may be made equal to T_1. Equation (5.6) then reduces to

$$h_1 - h_2 = A_T \ (P_1 - P_2).$$

Thus A_T can be determined directly from one experiment. In case of liquids however, μ_h, the Joule-Thomson coefficient is mostly negative and $T_2 > T_1$. Thus the method used for steam cannot be conveniently applied.

To avoid experiment (b), (the heater experiment), one of the coefficients, A_T or C_p for the liquid, may be determined by static experiments. Then the other property may be calculated directly from the results of experiment (a).

The above methods provide a simple and cheap means whenever a quick and direct assessment of the average properties, A_T and C_p is needed.

Provisional investigations have to be made for each case on its own merit, before it is determined whether to use these dynamic methods, or the static methods or whether to resort to a compromise as mentioned above, because the economy of the methods is largely dependent on the type of liquid, and the accuracy needed.

APPENDIX (5.1)

SAMPLE OF CALCULATION OF THE AVERAGE COEFFICIENT OF PRESSURE ENTHALPY

Consider the average isothermal coefficient of pressure enthalpy A_T.

$$A_T = \frac{1}{P-P_a} \int_{P_a}^{P} a_T \, dp. \qquad A.1.1$$

Assume P = 200 atm. and $\bar{T}$ = 15°C.

In Table 5.4 there are 9 (or 2n + 1) values of a_T at '2n' equal intervals of pressure i.e. dp = 25 atm. By Simpson's Rule and equation A.1.1.: (for convenience a_T is written as a),

$$A_T = (1/6n)(a_1+a_{2n+1})+(1/3n)(a_3+a_5+ \ldots +a_{2n-1}) + (2/3n)(a_2+a_4+ \ldots +a_{2n}) \qquad (A.1.2)$$

For eight equal intervals (i.e. n = 4),

$$A_T = 0.041667\,(a_1+a_9) + 0.083333\,(a_3+a_5+a_7) + 0.16667\,(a_2+a_4+a_6+a_8) \qquad (A.1.3)$$

The subscrips 1, 2, 3, 4 etc. refer to the pressures 1, 25, 50, 75 etc. respectively.

Thus by (A.1.3) $A_T = 0.94871$ ml^3/gm

at 15°C and 200 atm.

This is the value of A_T which is shown in Table 5.6 as A_T' or A_T/v_o.

Equation (A.1.2) above can be applied to any even number of intervals or strips. A_T for an odd number of strips is obtained by parabolic interpolation (or extrapolation) as following:

e.g. $a_2 = (3/8)\ a_1 + (3/4)a_3 - (1/8)a_5$,

$a_4 = (3/4)a_3 + (3/8)a_5 - (1/8)a_1$,

or by parabolic extrapolation,

$a_4 = 3(a_3 - a_2) + a_1$.

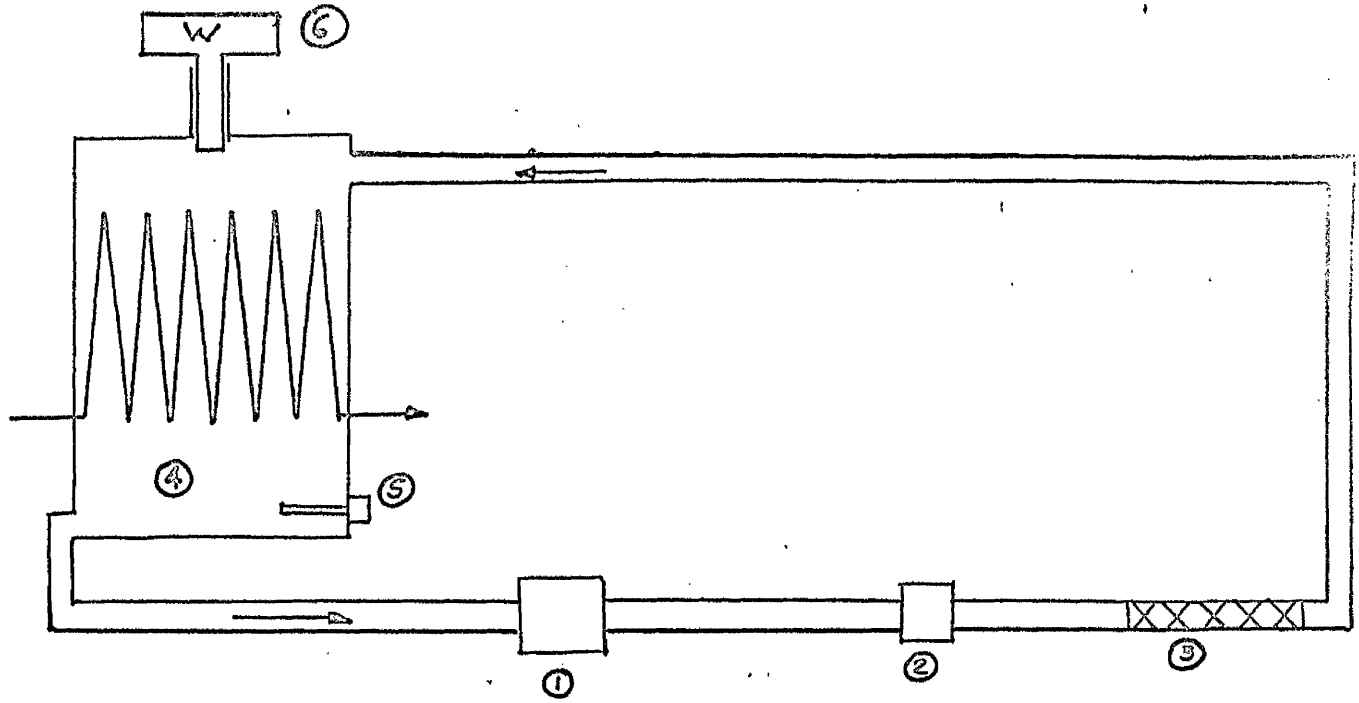


FIG. (5.1). *Apparatus for determining* C_p, A_t *and* $(\partial T/\partial P)_h$ *of liquids*

(1) *pumping set*
(2) *heating element for* C_p *experiment or throttle-valve for* $(\partial T/\partial P)_h$ *experiment*
(3) *throttle-valves or orifices*
(4) *thermostatic tank*
(5) *heater (for thermostatic control)*
(6) *pressure ram (for measurements at pressures higher than those of the pump)*

TABLE 5.1

v/v_o for water (adapted from Dorsey)

v_o = 1 ml/gm or 1/62.43 $ft^3/16_m$ One atmosphere = 1.0332 Kg_f/Cm^2 or 14.696 lb_f/in^2

(Ref. 6)

T °C / P Atmospheres	0	2	4	5	10	15	20	30	40	50
1	1.00013	1.00001	1.00000	1.00001	1.00027	1.00087	1.00177	1.00434	1.00770	1.01195
25	.99887	.99875	.99875	.99878	.99907	.99968	1.00059	1.00316	1.00658	1.01081
50	.99758	.99749	.99749	.99754	.99784	.99848	.99940	1.00198	1.00545	1.00967
75	.99631	.99624	.99624	.99633	.99666	.99732	.99826	1.00088	1.00434	1.00855
100	.99506	.99498	.99498	.99513	.99549	.99618	.99713	.99977	1.00322	1.00743
125	.99383	.99378	.99381	.99394	.99433	.99505	.99601	.99869	1.00213	1.00637
150	.99261	.99258	.99263	.99275	.99318	.99392	.99490	.99760	1.00103	1.00530
175	.99139	.99138	.99146	.99157	.99203	.99280	.99380	.99653	.99998	1.00424
200	.99018	.99018	.99028	.99040	.99089	.99169	.99271	.99545	.99892	1.00317

TABLE 5.2.

$10^5 \times (\partial v/\partial T)_p / v_o$

for water

T °C → / Atmosphere ↓	0	2	4	5	10	15	20	30	40	50
1	-6.62	-3.29	0.015	1.59	8.60	15.0	20.6	29.7	38.1	46.1
25	-5.94	-2.87	0.20	2.29	9.00	15.5	20.7	30.0	38.2	45.9
50	-5.10	-2.34	0.42	3.04	9.40	15.6	20.8	30.3	38.5	45.8
75	-4.32	-1.79	0.74	3.83	9.90	16.0	20.9	30.4	38.4	45.5
100	-3.45	-1.17	1.11	4.63	11.0	16.4	21.0	30.5	38.3	45.5
125	-2.53	-0.44	1.65	5.29	11.4	16.8	21.2	30.6	38.4	45.5
150	-1.77	+0.29	2.35	6.14	11.7	17.2	21.3	30.7	38.5	45.5
175	-0.98	+1.26	3.50	6.91	12.3	17.7	21.5	30.9	38.6	45.4
200	-1.58	+2.10	5.78	7.50	12.9	18.2	21.8	31.1	38.6	45.3

TABLE 5.3

$10^5 \times T \left(\partial v/\partial T\right)_p / v_o$ for water

T °C → / P ↓ Atmosphere	0	2	4	5	10	15	20	30	40	50
1	-1809	-905	+ 4	442	2436	4323	6040	9005	11933	14900
25	-1623	-790	55	637	2549	4467	6069	9096	11967	14803
50	-1393	-644	116	846	2662	4496	6099	9187	12058	14803
75	-1180	-493	205	1066	2804	4611	6128	9217	12027	14803
100	- 943	-322	308	1288	3115	4726	6157	9248	11996	14803
125	- 691	-121	457	1472	3228	4842	6216	9278	12027	14803
150	- 484	+ 80	651	1708	3313	4957	6245	9308	12058	14803
175	- 268	+347	970	1922	3483	5101	6304	9369	12090	14673
200	+ 432	+578	1602	2087	3653	5245	6392	9430	12090	14641

TABLE 5.4.

$$a_T^P = \left[v - T \left(\partial v / \partial T \right)_p \right] / V_o$$ for water

T °C / P Atmospheres	0	2	4	5	10	15	20	30	40	50
1	1.01821	1.00906	1.00000	.99599	.97591	.95764	.94137	.91429	.88837	.86295
25	1.01510	1.00665	.99820	.99241	.97358	.95501	.93990	.91220	.88694	.86246
50	1.01151	1.00393	.99633	.98908	.97122	.95352	.93841	.91011	.88487	.86164
75	1.00811	1.00117	.99419	.98567	.96862	.95121	.93698	.90871	.88407	.86052
100	1.00449	.99820	.99190	.98225	.96434	.94892	.93556	.90729	.88326	.85940
125	1.00074	.99499	.98924	.97922	.96205	.94663	.93385	.90591	.88186	.85834
150	.99745	.99178	.98612	.97567	.96005	.94435	.93245	.90452	.88045	.85727
175	.99407	.98791	.98176	.97235	.95720	.94179	.93076	.90284	.87908	.85751
200	.99450	.98440	.97426	.96953	.95436	.93924	.92879	.90115	.87802	.85676

TABLE 5.5.

The Average Isentropic Coefficent of Pressure Enthalpy, A_S', for Water

$$A_S' = \frac{A_S}{v_o} = \frac{1}{v_o(P_1 - P_2)} \int_{P_2}^{P_1} \left(\frac{\partial h}{\partial p}\right)_S dp$$

$$\left(\frac{\partial h}{\partial p}\right)_S = v$$

$$v_o = 1 \text{ ml/gm or } \frac{1}{62.4} \text{ ft}^3/\text{lb}_m$$

One atmosphere = 1.0332 Kg_f/cm^2 = 14.496 lb_f/in^2

T °C → / P Atmospheres ↓	0	2	4	5	10	15	20	30	40	50
1	1.00013	1.00001	1.00000	1.00001	1.00027	1.00087	1.00177	1.00434	1.00770	1.01195
25	0.99950	0.99938	.99938	.99940	.99967	1.00028	1.00118	1.00376	1.00714	1.01138
50	0.99886	0.99875	.99875	.99878	.99906	.99969	1.00062	1.00319	1.00658	1.01082
75	0.99822	0.99813	.99812	.99816	.99846	.99910	1.00003	1.00261	1.00602	1.01025
100	0.99759	0.99750	.99749	.99756	.99786	.99850	.99946	1.00204	1.00546	1.00969
125	0.99696	.99687	.99687	.99695	.99727	.99793	.99889	1.00146	1.00490	1.00912
150	0.99636	.99626	.99627	.99636	.99668	.99736	.99832	1.00091	1.00435	1.00858
175	0.99576	.99565	.99568	.99577	.99610	.99679	.99775	1.00036	1.00380	1.00804
200	0.99511	.99504	.99506	.99516	.99552	.99621	.99717	.99981	1.00326	1.00749

TABLE 5.6

The Average Isothermal Coefficient of Pressure Enthalpy, $A_T^{\prime}$, for Water

$$A_T^{\prime} = \frac{A_T}{v_o} = \frac{1}{v_o(P_1 - P_2)} \int_{P_2}^{P_1} \left(\frac{\partial h}{\partial p}\right)_T dp$$

$$\left(\frac{\partial h}{\partial p}\right)_T = v - T \left(\frac{\partial v}{\partial T}\right)_P$$

$$v_o = 1 \text{ ml/gm or } \frac{1}{62.4} \text{ ft}^3/\text{lb}_m$$

One atmosphere = 1.0332 Kg_f/cm^2 = 14.696 lb_f/in^2

T C° → / P ↓ Atmospheres	0	2	4	5	10	15	20	30	40	50
1	1.01822	1.00906	1.00000	.99599	.97591	.95764	.94137	.91429	.88837	.86295
25	1.01666	1.00787	.99913	.99421	.97478	.95637	.94064	.91321	.88759	.86273
50	1.01503	1.00661	.99820	.99246	.97358	.95521	.93991	.91221	.88684	.86241
75	1.01333	1.00528	.99721	.99074	.97231	.95426	.93918	.99129	.88612	.86199
100	1.01154	1.00387	.99618	.98906	.97096	.95321	.93844	.91045	.88545	.86146
125	1.00973	1.00243	.99511	.98741	.96945	.9212	.93769	.90970	.88486	.86091
150	1.00797	1.00091	.99384	.98575	.96800	.95101	.93693	.90894	.88425	.86042
175	1.00626	.99931	.99237	.98408	.96661	.94988	.93616	.90817	.88362	.85999
200	1.00466	.99767	.99070	.98242	.96530	.94871	.93537	.90741	.88297	.85965

TABLE 5.7

The Average Specific Heat at Constant Pressure, C_p', for Water *

$$C_p' = \frac{C_p}{J} = \frac{1}{J\,(T_1 - T_2)} \int_{T_2}^{T_1} \left(\frac{\partial h}{\partial T}\right)_P dT$$

$$\left(\frac{\partial h}{\partial T}\right)_p = C_p$$

$J = 426.6 \quad Kg_f m/Kcal \equiv 1400.2 \quad ft.\ lb_f/CHU.$

One atmosphere = $1.0332\ Kg_f/cm^2 \equiv 14.696\ lb_f/in.^2$

C° → / P ↓ Atmospheres	0	10	15	20	30	40	50
1	1.0078	1.0015	1.0000	0.9990	0.9982	0.9983	0.9988
25	1.0057	1.0006	0.9987	0.9973	0.9957	0.9959	0.9962
50	1.0041	0.9992	0.9974	0.9959	0.9942	0.9940	0.9941
75	1.0029	0.9982	0.9963	0.9948	0.9930	0.9927	0.9927
100	1.0021	0.9972	0.9954	0.9939	0.9922	0.9920	0.9918
125	1.0011	0.9963	0.9945	0.9930	0.9911	0.9905	0.9902
150	1.0001	0.9953	0.9934	0.9919	0.9898	0.9890	0.9886
175	0.9991	0.9941	0.9921	0.9905	0.9883	0.9875	0.9871
200	0.9981	0.9927	0.9906	0.9889	0.9867	0.9860	0.9856

* Adapted from Dorsey (Ref. 6)

PART B

TEMPERATURE MEASUREMENT

CHAPTER SIX

A SHORT SURVEY OF THE AVAILABLE APPARATUS, THE METHODS AND THE PROBLEMS OF TEMPERATURE MEASUREMENT IN FLOW SYSTEMS

6.1. Differential Thermometry

One of the major advantages of the version of the Steady Flow Energy Equation developed in Chapter 4, is that it reduces the need for accurate measurement of the absolute temperature to that for measuring a temperature difference, which is relatively much easier to obtain. Only an approximate estimate of the base absolute temperature is then necessary in order to determine the appropriate values of the thermodynamic properties A_s, A_T and C_p.

Differential measurements of temperature in flow systems are much easier in comparison with that of the absolute temperature, because the latter is always likely to be drifting. Observations of the temperature difference across a throttle valve showed that the difference was constant to within $\pm$ 0.5 m C^o for hours, inspite of the fact that the absolute temperature of the water was varying at the rate of 0.3 ^oC per hour, (provided, of course, that the rate of flow through the valve is constant). This stability of the

temperature difference depends mainly on the rate of change of the thermodynamic properties of the fluid with the change of temperature. In the case of water and most common liquids the effect of a temperature change of one or two degrees is negligible.

6.2 Choice of Thermometer

There are three main types of thermometers which may be used for the measurement of temperature difference.

(i) Thermocouples

These are sometimes attractive because of their sensitivity especially if a number of them are used in series and also since they indicate directly the temperature difference. They have, however, the disadvantage that their stability and hence accuracy is easily affected by contamination and strain. Furthermore great care is needed to ensure that no parasitic electromotive forces arise in the leads and connections. Thus in general thermocouples are not usually used where long leads are necessary.

(ii) Expansion Thermometers

Of these the most reliable and most accurate are probably the mercury thermometers, e.g. Beckmann

thermometers and the ordinary mercury-in-glass thermometers with fine graduations (e.g. 1/100 C^{o}). These are the simplest to use but they are fragile and need easy access to the points of measurment. They are most unsuitable when the measuring points are far apart and when there is the risk of breakage by vibrations as is the case in hydraulic systems. Moreover, since they do not indicate directly the temperature difference, they cannot be used unless the temperature of the fluid is steady and/or the temperature difference is very large and hence the resulting error is permissible.

(iii) Resistance Thermometers

Willm and Campmas, in their investigations of the practicability of the thermodynamic methods of measuring the hydraulic efficiency of turbines (Ref. 5) on behalf of Electricicite' de France concluded that platinum resistance thermometers are probably the most appropriate for this type of work. There are a number of reasons why these may be preferred to the other available types of thermometers.

(1) Their comparative robustness and stability.

(2) The possibility of siting the measuring points at a distance from each other without great inconvenience, especially on the development of simple lead compensation methods (Ref. 7) and Chapter 7.

(3) The experience acquired in high-precision resistance thermometry lead to standard methods for their manufacture, calibration, and to their adoption as the basis for the definition of the International Temperature scale (between the boiling point of oxygen (- 182.970 °C) and the melting point of antimony (630.5 °C).

For the above reasons resistance thermometers were used for all the precision thermometry required in this research. Two types of thermometers were used (i) The Barber-type 25 - Ω thermometers (Ref. 8) (ii) The commercial Platinum-in-Pyrex or Platinum-in-'Hartglass' thermometers.

The Barber-type thermometers used were manufactured by Messrs. Tinsley of South Norwood London. This is the standard type of thermometer used for the definition of The International Temperature Scale. It has a long glass stem (12" - 17") in which the platinum element is suspended in an atmosphere of helium. The thermometer has four leads which are joined to the element through leads made of gold to ensure long-time stability. The thermometers were calibrated in the National Physical Laboratory and were used in conjunction with the a.c. bridge which is referred to

below.

Since the standard thermometers described above are expensive and rather fragile their use was limited to the standardising tests only. For other tests commercial type thermometers were used. These consist of a 100 - Ω- Platinum, wire embedded in either 'Pyrex' or 'Hartglass' (hard glass). Some are supplied with nickel sheaths packed with 'alumina' powder. The others have to be fitted in a protective copper tube. The accuracy of these thermometers will be discussed in Chapter 7.

Resistance thermometers made of materials other than platinum may be used e.g. nickel and copper thermometers (Ref. 9), or thermistors which are made of semi-conductive materials with high resistance-temperature coefficients. However the stability of these thermometer-materials is much less guaranteed than that of platinum.

6.3 Measurement of temperature difference using resistance thermometers

Resistance thermometers are used in conjunction with ratio-bridges for the measurement of temperature difference. Most bridges for this use are of the d.c. type and are basically similar to the Wheatstone bridge

shown in Fig. (7.2). An example is the bridge used by Willm and Campnas (Ref. 5) which has standard resistors in its internal arms to give a ratio of 1:1. In addition it has a small potentiometer in series with one of these arms. This is used for the fine balancing of the bridge.

The bridge is mainly used with the 'null' method of measuring the hydraulic efficiency of turbines (Chapter 11). Here the function of the bridge is actually the matching of two thermometers at equal temperatures ($\Delta T = 0$) and not the measurement of a finite temperature difference. The zero-reading of the bridge is made to coincide with that of the galvanometer by adjustment of the potentiometer. If the bridge is to be used for measuring finite temperature differences, the potentiometer has to be calibrated against known temperature differences (e.g. across a throttle-valve).

Sometimes, instead of using a potentiometer the temperature difference is assessed from the galvanometer deflection (Ref. 10), the galvanometer having been calibrated in the same manner as the potentiometer. However it is well known that these bridges and measuring methods have a limited accuracy for the following reasons.

(i) The standard resistances and the resistance of the potentiometer are affected by the variations of the ambient temperature. To avoid this, the resistances may be kept in a thermostatic oil-bath; but this adds to the cost of the bridge.

(ii) These bridges may be reasonably accurate for measuring zero temperature differences, but, in the case of measuring finite temperature differences, the manipulation of the potentiometer may create thermal e.m.f.s. which limit the accuracy of the bridge. On the other hand the accuracy of the galvanometer deflection method is limited by the fact that the relation of the temperature difference with the deflection is not exactly linear, and by the accuracy with which the graduations of the galvanometer can be read. Furthermore, the fact that during measurement, the bridge is not actually at balance (because there is a small current passing through the glavanometer) invalidates the main advantage of using a Wheatstone-bridge circuit, i.e. the independence on the magnitude of the current of the main circuit and independence on the zero-position of the galvanometer spot or pointer. This latter condition makes it impossible to use a photo cell-amplifier galvanometer, because the zero position may always be drifting.

(iii) Great care has to be taken when long leads have to be used.

The above shortcomings of the available apparatus for temperature measurement prompted development work by the Electrical Engineering Department of The University of Glasgow on the a.c. transformer ratio bridge (Foord, Langlands & Binnie Ref. 7). This bridge, which is described briefly in section (7.4), is simple, very accurate and the defects inherent in the d.c. bridges described earlier are eliminated.

The a.c. transformer ratio-bridge was used by Dr. T.R. Foord, Mr. R.C. Langlands, and Mr. A.J. Binnie in collaboration with the author in much of the work reported in this thesis.

A simple and useful method for differential measurement of temperature by ratio-arm bridges was developed and successfully used. This is reported in detail in the following Chapter. It is particularly useful when the absolute temperature is likely to be drifting.

Another development was that of the hydraulic bridge. This is described in Chapter 9. It is a novel and useful device and is very simple to construct.

6.4 Main Sources of Errors in Measurement of Temperature in Fluid Flow Systems

The problems of temperature measurement described in the preceding sections are typical of the measuring instruments used. The following are the problems which are inherent in fluid flow systems and are likely to cause errors in the assessment of temperature difference irrespective of the accuracy of the measuring apparatus.

(i) Temperature drift (ii) temperature gradients in the pipe (iii) heat transfer (iv) the stagnation and frictional heating of the thermometer-pocket or the 'thermometer problem.'

The first three difficulties will be discussed briefly in the following sections because they have negligible effects in the case of the research work described here. The thermometer problem, however, will be discussed in detail in Chapter 10, because it had a significant effect on some of the tests.

6.5 (i) Temperature Drift

The drift of the absolute temperature of the flowing fluid is caused by two factors, (a) the variation of the temperature of the surroundings,

especially those of the feeding reservoir, (b) in close-circuit flow systems, i.e. where the same fluid is circulated in and out of the pump, the main effect is due to the transformation of a great part of the input energy of the pump into an internal energy of the fluid.

If the temperature change is not so large as to affect the properties of the fluid, the observed temperature difference will remain constant. However, due to the unsteady state of the incoming fluid, the temperature difference $(T_2 - T_1)$, measured instantaneously by thermometers placed at the inlet and outlet of the flow system, does not represent exactly the thermodynamic process taking place between the two measuring points (even in the absence of heat transfer). The error so involved is proportional to the time taken by the fluid to traverse the distance between these two measuring points. Because of the small sizes of the plants dealt with here, this time is relatively short and the phenomenon had negligible effects on the results obtained.

In certain cases the error due to this phenomenon may be significant, e.g. in the case of industrial water-turbines (Ref. 5). There the

distance between the measuring points may amount to more than one hundred feet and the water may take several seconds to cover the distance between the thermometer points. Furthermore vast variations of the temperature may occur due to the melting of the snow in the mountains round the reservoir and due to other climatic changes. In such cases a correction can be made by observation of the nature of the rate of change of the temperature through a long period.

6.6. Temperature Gradients in the Pipes

Heat transfer to the surroundings and the variation of the velocity in a pipe cause a transverse temperature gradient. This gradient is typical of the Reynolds and the Prandtl's numbers of the flowing fluid.

All the experimental work reported hereafter, was done with small pipes where it was possible to fit the thermometer pockets so that they spanned almost the whole diameter of the pipe and hence the mean-mixed or the bulk temperature of the fluid was obtained.

In the case of large pipes, measurements made by Thom and Ahmad (Ref. 11) on a 5' x 16' tail-race

of an industrial Pelton wheel revealed an appreciable temperature gradient. The authors reported a method which they had used for averaging the temperature based on drawing samples of water from different points of the section of the pipe to a common mixing chamber. They concluded that by placing the temperature sensor at a distance of D/7 from the wall, the error will amount to only 0.6%. Later Ahmad confirmed this figure analytically (Ref. 10).

A Bulletin by Messrs. Rosemount Engineering (Ref.12) gives interesting details of the suitable positions of different numbers of sensors for measurement of the average temperature of a flowing fluid having unity Prandtl number. The authors conclude, however, that if the temperature of the pipe is of the same order as that of the surroundings, the error involved by placing the temperature sensor anywhere in the pipe is negligible.

6.7 Heat Transfer

Consider the case of a fluid flowing inside an uninsulated horizontal metallic pipe. The heat exchanged between the fluid and the surroundings has to flow through a number of heat resistances:

(i) R_{h_1}, the resistance to heat transfer by conduction and forced convection between the fluid and the inside surface of the pipe.

(ii) R_{h_2}, the resistance to heat transfer through the pipe wall.

and (iii) R_{h_3}, the resistance to heat transfer by conduction and natural convection between the pipe and the surrounding air, and by radiation if high temperatures are involved.

If this external heat transfer is neglected, the error in the estimate of the temperature difference due to the actual thermodynamic process being treated may be represented by

$$\delta(\Delta T) = q/C_p = Q_e/C_p Q_m, \qquad (6.1)$$

$$\text{but} \quad Q_e = (L \times \Delta T_R) \; / \; (R_{h_1} + R_{h_2} + R_{h_3}), \qquad (6.2)$$

therefore

$$\delta(\Delta T) = (L \times \Delta T_R) \; / \; C_p Q_m \, (R_{h_1} + R_{h_2} + R_{h_3}) \qquad (6.3)$$

Apply equation (6.3) to the following typical installation:

Diameter of steel pipe = 5" thickness ≑ 1/4"

T, Water temperature = 16 C°. T_R, Room temperature = 21°C $T_R - T = 5°C$. (This is the maximum temperature difference which occurred during the experimental work.)

Distance between measuring points = 9 ft.

Typical Q for 5" ≑ D pipe = 400 g.p.m.

Calculation of R_{h_1}, R_{h_2} and R_{h_3}, shows that R_{h_1} is about 0.03% of R_{h_3}. R_{h_2} is also of the same order as R_{h_1}. Thus R_{h_1} and R_{h_2} may be neglected and equation (6.3) may be modified to the form:

$$\delta(\Delta T) = (L \times \Delta T_R) / (C_p Q_m R_{h_3}) \qquad (6.4)$$

$$R_{h_3} = (1 / \pi D h_c), \qquad (6.5)$$

where h_c is the coefficient of heat transfer by conduction and natural convection between the pipe and air. This is a function of Grashof and Prandtl numbers. From Fishenden and Saunders (Ref. 13), h_c = 0.54 CHU/ft.^{2} hr. C°. Thus $\delta(\Delta T)$ is equal to about 0.15 m°C.

For pipes with concrete and other materials surrounding them the error due to heat transfer is likely to be less than that calculated above. For large pipes the area exposed to heat transfer is large but Q_m

is also large. Thus for most normal conditions and for fluids at atmospheric temperatures, the effect of heat transfer is negligible.* It is always useful, however, to make an approximate estimate of the effect of heat transfer. For most conditions equation (6.4) gives a useful guide to the order of this error.

Certain precautions have to be made regarding the design and location of the pocket to avoid direct heat transfer from the surroundings to the thermometer. Fig. 6.1 shows a design of a typical pocket used. The actual pocket is made of a copper tube (usually of 5/8" D and 1/16" thickness) filled with water. The tube is completely insulated from the metallic fittings by Tufnol bushes. The pocket can easily be fitted, adjusted and dismantled.

It is always preferable to locate the pocket where it cannot be directly affected by forced currents of air from fans or ventilators. If this is not possible an appropriate shelter must be provided.

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* This was confirmed practically by Test No. 2, Chapter 11.

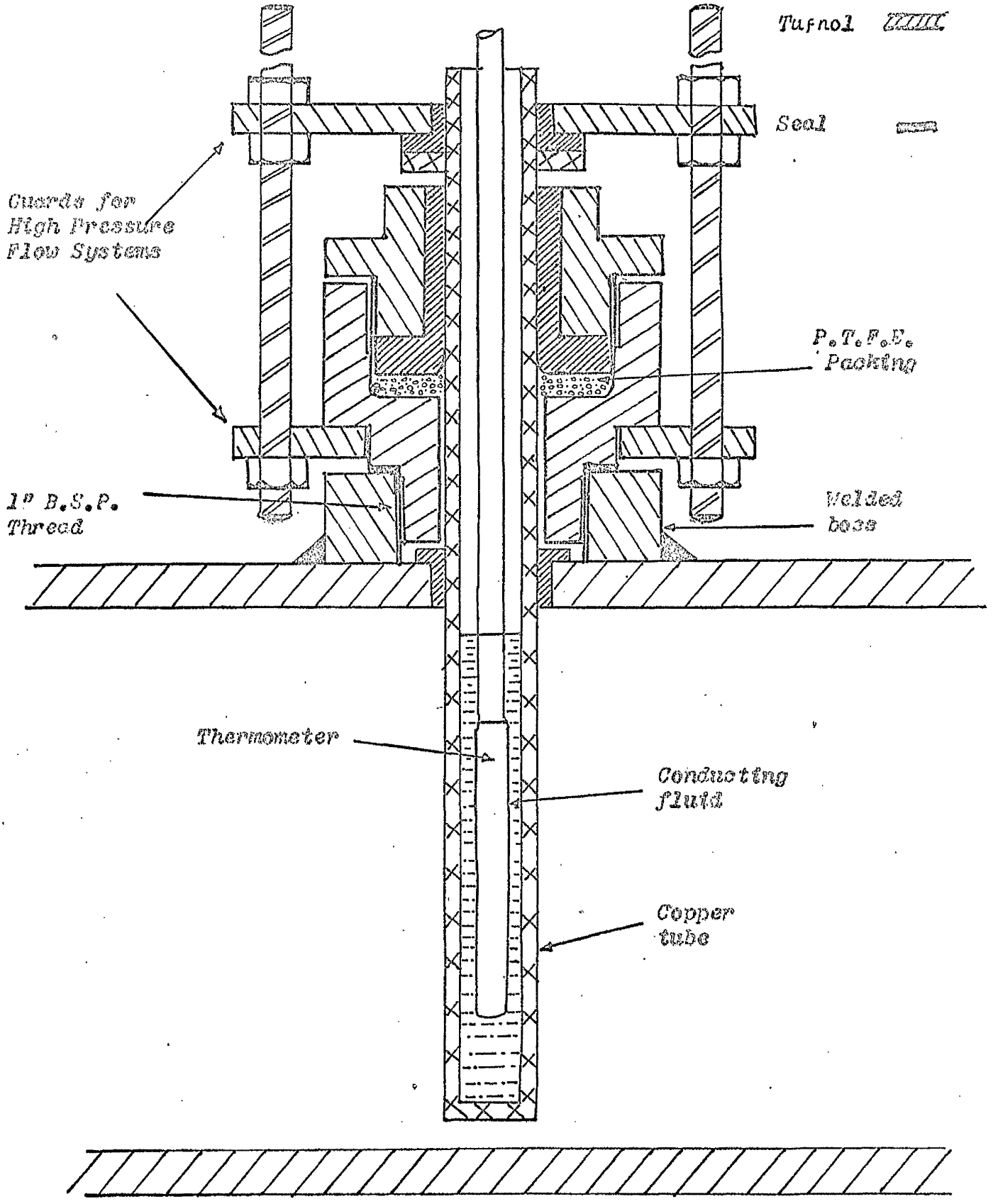

FIG. 6.1 ADJUSTABLE THERMOMETER-POCKET

CHAPTER SEVEN
THE PAIR RATIO METHOD OF MEASURING SMALL TEMPERATURE DIFFERENCE *

7.1. INTRODUCTION: THE PAIR RATIO

The relation between resistance and temperature of platinum is usually expressed by either of the following equations:

$$R_T = R_o(1 + a\bar{T} + b\bar{T}^2) \qquad (7.1)$$

$$\bar{T} = \frac{R_T - R_o}{\alpha_o R_o} + \delta(\bar{T} - 100)\bar{T} \times 100^{-4} \qquad (7.1.a)$$

The notation $\bar{T}$ indicates that the Celsius temperature is to be used.

Correlating equations (7.1) and (7.1.a),

$$a = \alpha_o\left(1 + \frac{\delta}{100}\right) \qquad b = \frac{-\alpha_o\delta}{100^2}$$

Typical values of these coefficients for platinium are

$$\alpha_o = 3.925 \times 10^{-3} \qquad a = 3.984 \times 10^{-3}$$
$$\delta = 1.493 \qquad b = -5.86 \times 10^{-7} \qquad (7.2)$$

The pair ratio $(K_P)_T$ of two resistance thermometers is defined here as the ratio of their resistances when both are placed at the same reference temperature, $\bar{T}$. Denoting the two thermometers by the letters x and y, from equation (7.1)

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* Elagib, J. of Scientific Instruments (Ref. 14)

$$(K_P)_{\bar{T}} = \frac{(R_y)_{\bar{T}}}{(R_x)_{\bar{T}}} = \frac{(R_y)_o}{(R_x)_o} \frac{(1 + a_y\bar{T} + b_y\bar{T}^2)}{(1 + a_x\bar{T} + b_x\bar{T}^2)} \qquad (7.3)$$

If the two thermometers have identical temperature coefficients a & b, the pair ratio, K_p, will be independent of the value of $\bar{T}$. However it is hardly possible in practice to manufacture thermometers of identical temperature coefficients due to the inevitable variation of the purity of the metal of the wires. Nevertheless, if the thermometers have matched coefficients or if they are manufactured to the same standard specification, the variation of K_P with $\bar{T}$ is very small because $a_y \doteq a_x$ and $b_y \doteq b_x$. The theoretical and practical verification of this will be discussed later in section 7.5. with reference to Appendix (7.1.)

The stable nature of the pair ratio makes its measurement very simple as no exact maintenance of the reference temperature is necessary. It also need not be redetermined unless there is substantial change of the reference temperature (section 7.5), or if the thermometers are subjected to a shock and it may be suspected that their R_o values are changed.

7.2. THE TEST RATIO AND THE MONITOR RATIO

Consider two values of $\bar{T}$, namely, $\bar{T}_1$ & $\bar{T}_2$ which are the temperatures at two sections of a flow system, e.g. at inlet and outlet of a hydraulic machine or a heat exchanger. If the two thermometers are both placed at the temperature $\bar{T}_1$. (section 7.5) their pair ratio may be determined by obtaining balance with a suitable ratio-arm bridge (refer to figs. (7.1) and (7.2) and section 7.4)

The thermometer, y is chosen as monitor and is transferred to $\bar{T}_2$. The new ratio K_t is noted:-

$$K_t, \text{ The test ratio} = \frac{(R_y)_{T_2}}{(R_x)_{T_1}} \qquad (7.4)$$

Dividing the test ratio by the pair ratio, of equation (7.3), the monitor ratio is obtained.

$$K_m, \text{ The monitor ratio} = \frac{K_t}{(K_P)_{T_1}} = \frac{(R_y)_{T_2}}{(R_y)_{T_1}} \qquad (7.5)$$

Substituting for $(R_y)_{T_2}$ & $(R_y)_{T_1}$ from eqn(7.1)

$$\text{The monitor ratio, } K_m = \frac{(1 + a_y\bar{T}_2 + b_y\bar{T}_2^{\,2})}{(1 + a_yT_1 + b_yT_1^{\,2})} \qquad (7.6)$$

Thus by using the pair ratio it is possible to obtain a ratio, K_m, which is a function of the two temperatures $\bar{T}_1$ & $\bar{T}_2$ and the temperature coefficients of the monitor thermometer, y. Km does not include the resistance parameter $(R_y)_o$ or any parameter of the thermometer x, which acts as a reference only.

7.3. DETERMINATION OF THE TEMPERATURE DIFFERENCE, T_2-T_1.

To determine the temperature difference (T_2-T_1) between inlet and outlet sections of the flow system, applying Taylor series to eqn. (7.1)

$$\delta R_t = R_o\ (a + 2b\bar{T})\ \delta T + Ro\ (2b)\ \frac{(\delta T)^2}{2}$$

Substituting $\bar{T}_1$ for $\bar{T}$, $(T_2 - T_1)$ or ΔT for δT and

$(R_{T_2} - R_{T_1})$ for δR_T and rearranging

$$R_{T_2} - R_{T_1} = R_o\ (a + 2\ b\bar{T}_1 + b\,\Delta T)\ \Delta T. \quad (7.7)$$

Dividing eqn. (7.7) by $R_{T_1} = R_o\ (1 + a\bar{T}_1 + b\bar{T}_1^{\,2})$ and rearranging,

$$\Delta T = \left(\frac{R_{T_2}}{R_{T_1}} - 1\right) \frac{(1 + a\bar{T}_1 + b\bar{T}_1^{\,2})}{(a + 2bT_1 + b\Delta T)} \quad (7.8)$$

From eqn. (7.5) $\dfrac{R_{T_2}}{R_{T_1}} = Km.$ From eqn. (7.2) it is seen that b is very small compared with a. Also since ΔT

is small (eg $< 10\ C^{o}$) b ΔT is only a fraction of one percent of a and can be neglected. So, eqn. (7.8) may be re-written as

$$\Delta T = (K_m - 1)\ \frac{(1 + a_m \bar{T}_1 + b_m \bar{T}_1^{\,2})}{(a_m + 2b_m \bar{T}_1)} \qquad (7.9)$$

The subscript (m) refers to the monitor thermometer.

If the coefficient (b) is neglected eqn (7.9) reduces to

$$\Delta T = (K_m - 1)\ \left(\frac{1}{a_m} + \bar{T}_1\right) \qquad (7.10)$$

Eqn. (7.10) can be used for platinum if $\bar{T}_1$ is small i.e. when $b_m \bar{T}_1^{\,2}$ and $2b_m \bar{T}_1$ may be neglected.

The value of $\frac{1}{a_m}$ in eqn. (7.10) is usually about $250^{o}C$. Thus $\bar{T}_1$ can be measured very approximately and no serious error will be incurred in the estimation of ΔT, e.g. an error of $2.5\ C^{o}$ in the measurement of $\bar{T}_1$ will cause an error of less than 1% in the estimation of ΔT.

On the other hand an error of 4×10^{-6} in the estimation of K_m induces an error of $1\ m^{o}C$. From this relation the number of the bridge decades necessary for a certain limit of accuracy may be determined.

It is seen from eqn. (7.5) that K_m is a quotient of two ratios (K_t/K_p) which are almost equal. Thus all relatively constant errors, e.g. errors due to

inequality of leads in d.c. bridges, are completely or almost completely cancelled out, provided that there is no substantial variation of the ambient temperature during the time interval since measurement of the pair ratio. Nevertheless it is estimated that 0.1 Ω inequality of lead resistances, in a d.c. circuit containing 100 Ω thermometers, will induce only 1% error in the measurement of a temperature difference of 1 C^{o}, even if the ambient temperature varies by 10 C^{o}. This is estimated from the approximate relation.

$$\delta(\Delta T) \doteq \frac{\delta l}{R} \times \frac{dT}{R} \qquad (7.11)$$

7.4. RATIO-ARM BRIDGES

The a.c. transfermer ratio-bridge shown in fig. (7.1) is described in detail by Foord, Langlands and Binnie* and is referred to in section 7.5. It incorporates a precise 6-decade ratio transformer and two separate toroidal transformers for lead compensation. The bridge is capable of measuring temperature and temperature difference to a high degree of accuracy e.g. of the order of ± 1 mC^{o}.

For less accurate measurements d.c. bridges of the form shown in fig. (7.2) can be used.

* Ref. 7

Their accuracy is greatly enhanced if such a differential method like the pair ratio is used (section 7.3). Errors due to the thermometer leads are eliminated by using 4-core leads. A pair is connected to the thermometer and a dummy pair is connected in series with the other thermometer (as in the Callendar-Griffiths bridge). Thus the two thermometers do not only have equal leads, but the leads are at the same temperature throughout their length. Strictly, the resistances of the leads must be in the ratio of the resistances of the thermometers before the lead errors are completely eliminated.

$$\frac{R_y}{R_x} = \frac{l_y}{l_x} = \frac{R_y + l_y}{R_x + l_x} \qquad (7.12)$$

Thus the thermometers used must be - as nearly as possible - of equal R_o values. The effect of the leads may be diminished if the ratios R_y / l_y and R_x / l_x are high.

If the internal resistances of the bridge are kept under controlled temperature a high degree of accuracy, of the order of milli degrees is obtainable.

With both these types of bridges a photo-cell amplifier galvanometer can be used to increase the

sensitivity and accuracy of measurements. This is a significant advantage of the pair ratio method over the methods where the deflection of the galvanometer is used to indicate the magnitude of the temperature difference, because, as its electrical zero needs occasional re-adjustment, the amplifier galvanometer is only most accurate for null measurements.

Simple methods of calculating the temperature difference from the ratio measurements made by these a.c. and d.c. bridges are shown in Appendixes (7.2) and (7.3.) respectively.

7.5. PRACTICAL MEASUREMENTS

A simple method for obtaining the pair ratio is to place the thermometers in adjacent pockets in the flowing fluid. This was found to give very steady and reproducible results to within less than $\pm$ 1 x 10^{-6}. The only precautions necessary were to have identical pockets and reasonably turbulent flow.

Practical measurements with a selected pair of standard thermometers of the Barber design* showed

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* The platinum wire in these thermometers is kept in an atmosphere of helium and connected with golden leads. It is the standard design adopted by NPL, (The National Physical Laboratory). (Ref. 8)

that the same pair ratio at triple-point-cell temperature (i.e. 0.010 C^{o}) was reproducible to the sixth decimal place at 15°C.

The variation of K_p with temperature may be represented approximately by the equation,

$$dK_p = \frac{(R_y)_o}{(R_x)_o} \frac{(a_y - a_x)}{(1 + a_x T)^2} dT \qquad (7.13)$$

which is obtained by differentiating eqn (7.3) and neglecting the effect of b (Appendix 7.1).

Thus a steady pair ratio, in spite of the variation of the fluid absolute temperature, indicates that the thermometers have equal or nearly equal "a" values and hence may be considered as matching.

dK_p was calculated (from information on the N P L Certificates) for a number of randomly selected standard thermometers of the Barber type. It was found not to exceed 1×10^{-6}, for a temperature change of 1 C^{o}.

Experiments carried out on commercial Platinum-in-pyrex 100 thermometers, using water at room temperatures, have shown that they would hold true their pair ratio to within $\pm 2 \times 10^{-6}$ for several days provided that variation of the water temperature was within $\pm$ 1 C^{o}.

It may be useful in practice however to prepare beforehand (by calculation or by measurement) a table of the pair ratios for the possible range of fluid temperatures at which the experiments are carried out. This will obviate the need for transferring the monitor thermometer during tests.

The pair ratio method was found to give results identical with those obtained by the reversing method described by Foord, Langlands & Binnie and later applied for the measurements described by Foord, Langlands and El Agib in ref 15 and in chapter 8). In these measurements the a.c. bridge shown in fig (7.1) was used to measure the small temperature rise across an orifice in a water pipe. The temperature differences measured ranged between 0.016 to 0.08 C^{o}. They were found to agree to within $\pm$ 0.001 C^{o} with those obtained by calculations based on the pressure drop (eqn 8.1).

The main advantage of the pair ratio metod is that no transfer or reversing of the thermometers during testing is necessary. At worst only two pair ratio readings are needed: one at the beginning and one as a check at the end of a series of tests.

It must be emphasized that the pair ratio method is only a method for measurement of the difference of the actual temperatures of the thermometer-elements at the instant of measuring. All necessary precautions must be made to prevent or correct for the errors due to heat transfer, stagnation and temperature gradients etc.

7.6. CONCLUSIONS

It is well established that, for accurate measurements it is futile to attempt to determine temperature difference in flow systems by measuring the temperature at one end and then the other; because although the difference of the temperatures is of static nature their absolute values are not.

The pair ratio method provides a simple means for direct differential measurements. It simplifies the use of resistance thermometers, which by virtue of their stability ($<630^{\circ}C$) and the existance of relatively more accurate means of their calibration are much preferred to other types of temperature sensing elements.

The method does not require knowledge of the R_o values of the thermometers; and since it is based entirely on differential ratio measurements, cheap and simple ratio-arm bridges can be used with greater accuracy.

APPENDIX (7.1)

VARIATION OF THE PAIR RATIO WITH TEMPERATURE

If the coefficient b is neglected, eqn 7.3 may be re-written as

$$K_p = \frac{(R_y)_o}{(R_x)_o} \quad \frac{(1 + a_y\bar{T})}{(1 + a_xT)} \qquad (A.1.0)$$

Differentiating,

$$dK_p = \frac{(R_y)_o}{(R_x)_o} \left[\frac{a_y}{1 + a_xT} - \frac{a_x(1 + a_y\bar{T})}{(1 + a_xT)^2} \right] dT$$

$$= \frac{(R_y)_o}{(R_x)_o} \left[\frac{(a_y - a_x)}{(1 + a_xT)^2} \right] dT \qquad (A.1.1)$$

The maximum variation occurs at $\bar{T} = o$,

$$dK_p = \frac{(R_y)_o}{(R_x)_o} \times (a_y - a_x)\ dT. \qquad (A.1.2)$$

APPENDIX (7.2)

Measurement of small temperature difference by the pair ratio method and using an a.c. transformer ratio-bridge

Simple method for calculation of ΔT

From the theory of the transformer ratio-bridge, the balance ratio obtained, n, is equivalent to the ratio of the resistance of a thermometer to the sum of the resistances of the two thermometers, e.g.

$$n = R_y / (R_y + R_x) \qquad (A.2.0)$$

(a) thermometer x is used as monitor

let $k = R_x/R_y$

$$\therefore \quad k = (1/n) - 1 \qquad (A.2.1)$$

From eqn.(7.5) $\quad K_m = K_t/K_p$

thus in eqn (7.9) and (7.10),

$$K_m - 1 = (K_t/K_p) - 1 = (1/K_p)(K_t - K_p) \qquad (A.2.2)$$

The subscripts m, t, and p refer to the monitor, the test and pair ratios respect.

the differentiation of eqn (A.2.1) gives

$$dK = -(1/n^2)dn \qquad (A.2.3)$$

Denote the test and the pair ratios obtained at the bridge by n_t and n_p respectively. These correspond to K_t and K_p. Thus by equation A.2.3.

$$K_t - K_p = -(1/n_p^2)(n_t - n_p) \qquad (A.2.4)$$

Thus from equations (A.2.2) and (A.2.4)

$$K_m - 1 = (1/K_p n_p^2)(n_p - n_t) \qquad (A.2.5)$$

by substituting for K_p from equation (A.2.1), equation (A.2.5) becomes,

$$K_m - 1 = \frac{(n_p - n_t)}{n_p\,(1 - n_p)} \qquad (A.2.6)$$

For most of the platinum in pyrex or platinum in glass thermometers used in this research,

$1/(n_p)(1 - n_p) \doteq 4.00$ to within $\pm$ 0.1%.

The values of $(K_m - 1)$ obtained by equation (A.26) can be used in equation (7.9) or (7.10). Calculation can thus be carried out using only a slide rule, otherwise (if K_t and K_p are to be obtained directly from values of n_t and n_p by equation (A.2.1)), a 12 - figure calculating machine has to be employed for accuracies of $\pm$ 1 mC°. Equation A.2.6 may be used for temperature differences of the order of $\pm$ 10 C° with only a fraction of a percent error.

(b) Thermometer 'y' is used as a monitor

The same foregoing procedure may be followed.

$$(1/K) = (R_y/R_x) = (1/n) - 1 \qquad (A.2.7)$$

Differentiation of equation (A.2.7) gives

$$dK/K^2 = dn/n^2$$

$$\therefore \quad dK = (K^2/n^2)\, dn = dn/(1-n)^2 \qquad (A.2.8)$$

By substituting $(K_t - K_p)$ and $(n_t - n_p)$ for dK and dn

$$(K_m - 1) = (K_t - K_p)/K_p = (n_t - n_p)/(1-n_p)^2\, K_p$$

$$= (n_t - n_p)/(1 - n_p)\, n_p \qquad (A.2.9)$$

APPENDIX (7.3)

Measurement of small temperature difference by the pair ratio method and using d.c. bridges

Simple method for calculation of ΔT

Let R_x, R_y refer to the resistance thermometers as before, and let (R_a), (R_b) refer to the corresponding resistance-boxes in the bridge (Fig. 7.2). R_x is used as monitor (R_a) is adjustable and R_b is fixed.

Let $K = (R_a)/(R_b)$ $\therefore$ $dK = dR_a/R_b$ (A.3.0).

From equation (A.2.2) in Appendix (7.2),

$$K_m - 1 = (1/K_p)(K_t - K_p) \qquad (A.3.1).$$

Using equation (A.3.0) and (A.3.1), and substituting for dK and dR_a by $(K_t - K_p)$ and $(R_a)_t - (R_a)_p$ respectively,

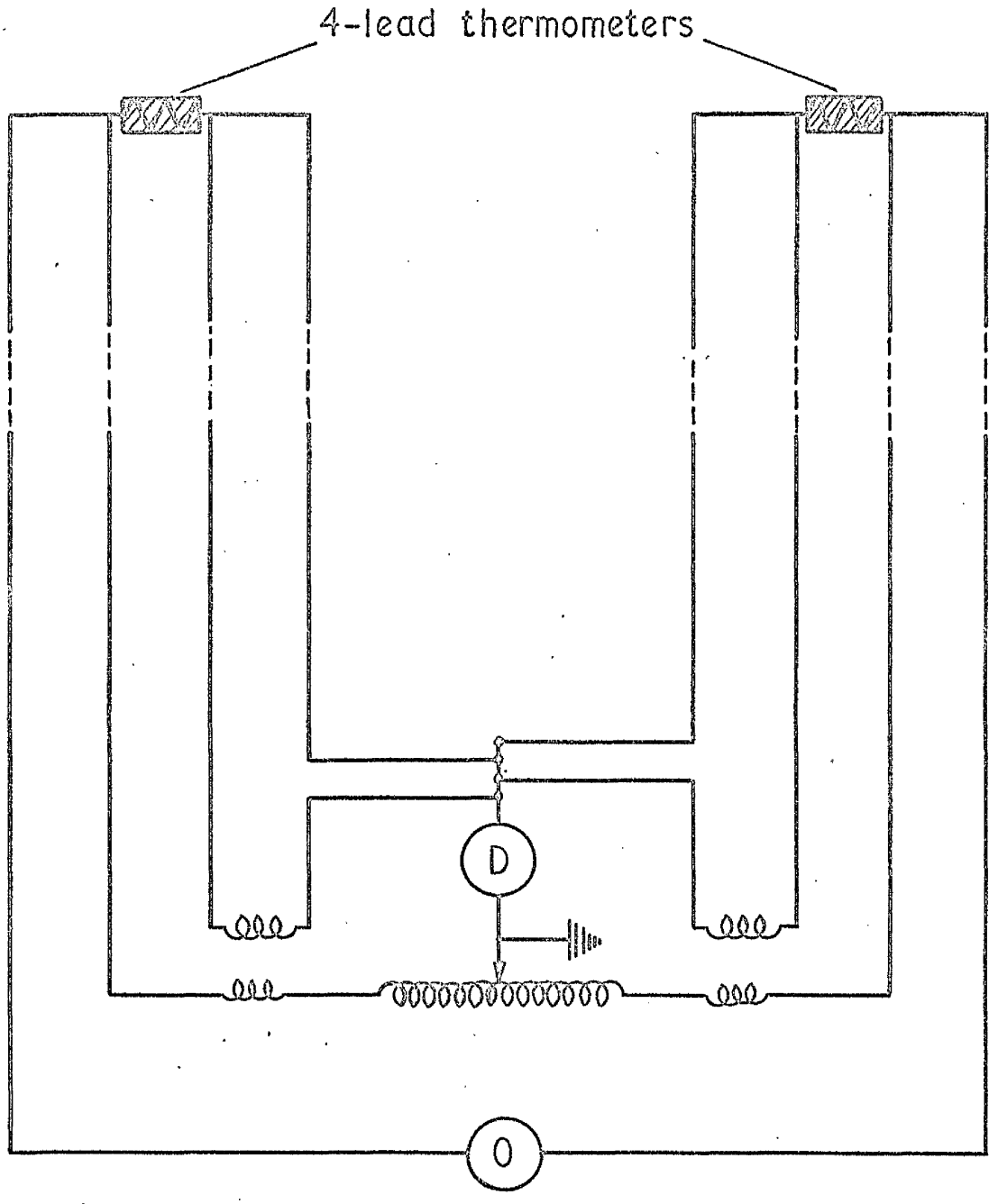
$$K_m - 1 = dR_a/R_b\ K_p$$

$$= \frac{(R_a)_t - (R_a)_p}{(R_a)_p} \qquad (A.3.2)$$

As before the subscripts m, t, and p refer to the monitor, test and pair ratios respectively.

Note that equation A.3.2 for $(K_m - 1)$ does not include R_b or the resistance of any of the thermometers. The only resistance which need to be known is (R_a).

FIG. 2.1. The a.c. transformer ratio-bridge (after Foord, Langlands and Binnie)



O Audio-frequency oscillator
D Tuned amplifier detector

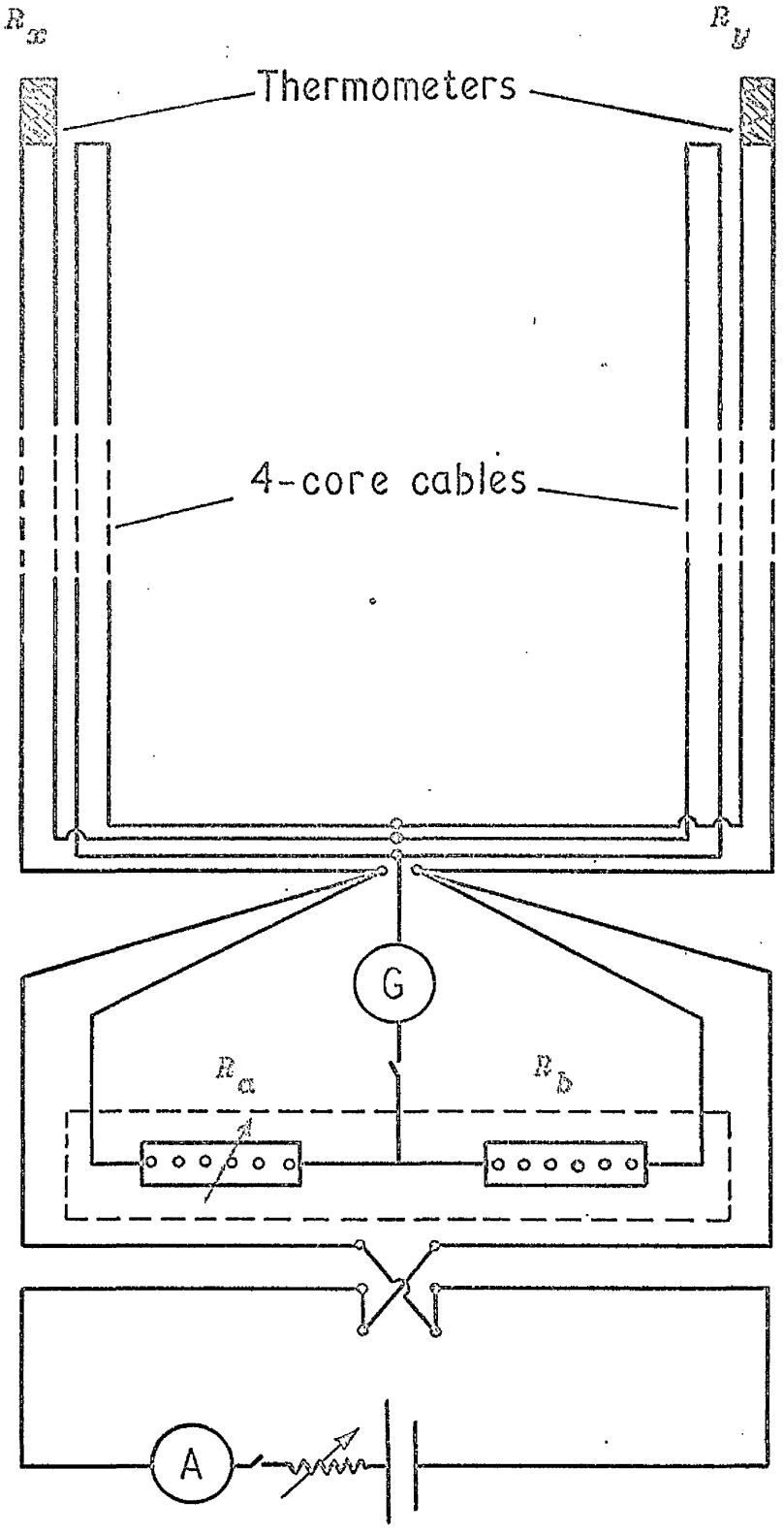


FIG. 7.2. d.c. Wheatstone bridge

CHAPTER EIGHT

A HYDRAULIC THROTTLING CALORIMETER FOR CHECKING AND CALIBRATION OF TEMPERATURE MEASURING APPARATUS

8.1 Introduction

An orifice or a throttle valve across a uniform horizontal pipe creates a pressure drop and a corresponding incremental change in the temperature of the fluid which can be correlated by the following equation:-

$$A_{T_1} (P_1 - P_2) + C_{P_2} (T_1 - T_2) = 0 \qquad (8.1)$$

This equation is obtained by the integration of equation (2.6) or from equation (4.14) with the assumption that $V_1 = V_2$ and $Z_2 = Z_2$.

As a guide to the order of temperature difference obtainable when water at room temperatures is used, one foot drop of head of water induces a rise of 0.7 $m^{o}C$ (i.e. 0.0007^{o}C). This high value of pressure/temperature relation makes it possible to calculate the temperature difference with great accuracy from only approximate pressure measurements. This form of hydraulic throttling calorimeter can be used as a

source of exact temperature differences against which temperature-measuring apparatus may be standardised.

The Hydraulic Throttling Calorimeter*

A general view of the throttling calorimeter is shown in Fig. (8.1). It consists of a uniform mild steel pipe, 5" i.d., placed horizontally and centrally in a wooden box 22" x 22" in cross section and 18' long. A brass plate with a $1\frac{5}{8}$" dia. orifice is flanged across the pipe in the middle of the box and the pipe is thermally insulated by Polyzote granules which fill the box. The insulation is designed according to Appendix (8.1) and the maximum heat Transfer error of < 0.5 mC°. Two thermometer pockets and two pressure measurement glands are symmetrically arranged on each side of the orifice. These access points are situated so that the distance between the pockets (approximately 9') is the same as that between the glands. The 4"/3" centrifugal pump described in Chapter 11 is used to supply the water flow through the calorimeter. By designing the orifice to match the characteristics of this pump (Refer to Fig. 8.2) a variable pressure difference is

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* Adapted from N.E.L. Report No. 130 by Foord, Langlands and Elagib.

obtained across the orifice as the pump speed is varied. In this way, the difference in head which can be obtained ranges from 25' to 140'. From the Joule-Thomson relation or from equation (8.1) the corresponding range of temperature difference existing across the orifice is approximately from 16 to 100 millidegrees Celsius. By using orifices of different sizes, this range could be extended and yet still ensure sufficient turbulence to give homogeneity of temperature within the pipe without setting up excessive vibrations.

A butterfly valve is fitted in the downstream section of the calorimeter and was used to bleed off air to ensure that the pipe is running full. The downstream pressure, P_2, is kept as near atmospheric as possible. This is done because the values of A_T' shown in Table (5.6) and used for calculations of temperature difference are based on the lower pressure being atmospheric. A simple water manometer is used to measure this downstream pressure. The upstream pressure, P_1, is measured by the Singh manometric piston gauge*, shown in the foreground of Fig. (8.1). As used in this work, this instrument permits measurements of head to be made to $\pm$ 0.04 ft. which corresponds, by use of equation (8.1), to an accuracy of $\pm$ 3 x 10^{-5} °C in the estimation of ΔT.

* (Ref. 16)

The ultimate accuracy obtainable by use of equation (8.1) is clearly dependent on the accuracy to which the values of A_{T_1} and C_{p_2} are known, and these quantities are functions of water purity. As there is little published information on the effect of impurities on the physical properties of water, it is advisable to use water free from air and as pure as possible.

The results of proving tests on the a.c. bridge using a pair of Barber-type thermometers and the pair ratio method are shown in Table No. (8.1). It will be seen that the temperature difference, ΔT, determined by the two quite different methods agree to within about 1 $m^{o}C$.

8.3 Calibration of resistance thermometers using the hydraulic throttling calorimeter *

In Chapter (7) it was shown that for measurement of temperature difference by the pair ratio method, knowledge of the coefficient of only one thermometer is necessary. The following method provides a simple means of determining the coefficient of resistance thermometers. The procedure is the same

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* Elagib, Journal of Scientific Instruments (Ref. 14) Oct. 1964, Pg. 592 Vol. 41.

as that described in section (7.3). The pair ratio and the test ratio are determined. From these the monitor ratio is calculated. The value of ΔT or $(T_2 - T_1)$ is calculated by equation (8.1). To be able to obtain the two temperature coefficients 'a' and 'b' of the monitor thermometer, a pair of different readings of ΔT has to be made. Two simultaneous equations similar to equation (7.9) are then formed from which a and b are determined.
To obtain an accurate value of 'b' a high pressure system has to be used.

However, this procedure can be avoided by using either of the following methods.

(1) The effect of the coefficient 'b' is neglected and equation (7.10) is used to determine a_m or strictly, and equivalent value of a_m, denoted by A_m.

$$A_m = \frac{K_m - 1}{\Delta T - (K_m - 1)\,\bar{T}_1} \qquad (8.2)$$

This equivalent value of a_m can be used for measurements with very satisfactory results provided that $\bar{T}_1$ in measurement is in the region of the calibration temperature. It can be proved that even if it is different by $\pm$ 10 C^o an error of only 0.3% of ΔT will be incurred. Results of calibration made by

this method are shown in Table (8.2).

(2) If calibration and measurements are not to be carried out at nearly the same temperature, then quation (7.9) has to be used. To reduce the number of unknowns an arbitrary value of b of-5.9 x 10^{-7} may be used for platinum. Fractional variation of this value of b from the actual value has an insignificant effect on the accuracy of the results (refer to (i) above for proportional estimation of the error).

It must be remembered that the ultimate accuracy of calibration is dependent on the magnitude of the pressure drop and the corresponding temperature change. The same percentage accuracy will be attainable when the thermometers are used for measuring. Thus the thermometers must not be calibrated against small temperature difference and used for measurements of large temperature difference unless the resulting inaccuracy is permissible.

Larger temperature differences are obtainable if fluids other than water are used, e.g. steam, provided their thermodynamic properties A_T and C_P are accurately known.

APPENDIX 8.1

DESIGN OF THERMAL INSULATION FOR THE 5"-D CALORIMETER PIPE

't' in. radial thickness of polyzote insulation	5"	6"	8"	10"	12"
$\delta(\Delta T)$, error due to heat transfer through insulation ($\pm$ mC°)	0.67	0.60	0.49	0.45	0.40
Weight of polyzote needed (approx) (lbs)	31	41	64	92	125
Cost of polyzote (£) (approx)	13.0.0	17.0.0	25.0.0	37.0.0	50.0.0

$\delta(\Delta T)$ was calculated by equation (6.3):

$$\delta(\Delta T) = (L \times \Delta T_R)/(C_p' Q_m (\Sigma R_h) \qquad (A.1.0)$$

Here the major resistance is that of polyzote

$$(R_h)_p = t / \pi k D_m \qquad (A.1.1)$$

D_m is the logarithmic mean diameter

$$= 2t / \log_e[(r_o + t)/r_o] \text{ (Ref.13)} \qquad (A.1.2.)$$

k, the thermal conductivity of polyzote

$$= 0.43 \text{ cHU}/\text{ft}^2 \text{ hr. } ^\circ\text{C/in.}$$

density of polyzote

$$= 1.0 \text{ lb/ft}^3$$

The other quantities in equation (A.1.0) are assumed as following

L (maximum) = 20 ft. ΔT_R (maximum) = $\pm$ 10 C^o

Q (minimum) = 120 g.p.m.

An allowance of 10% is made for losses other than those due to transverse conduction.

It was decided that the 8" thickness is the most suitable, for the following reasons:

(i) It is appropriate when considering the length of the available thermometers

(2) The improvement on the accuracy by using thicker insulation is negligible and does not warrant the extra cost.

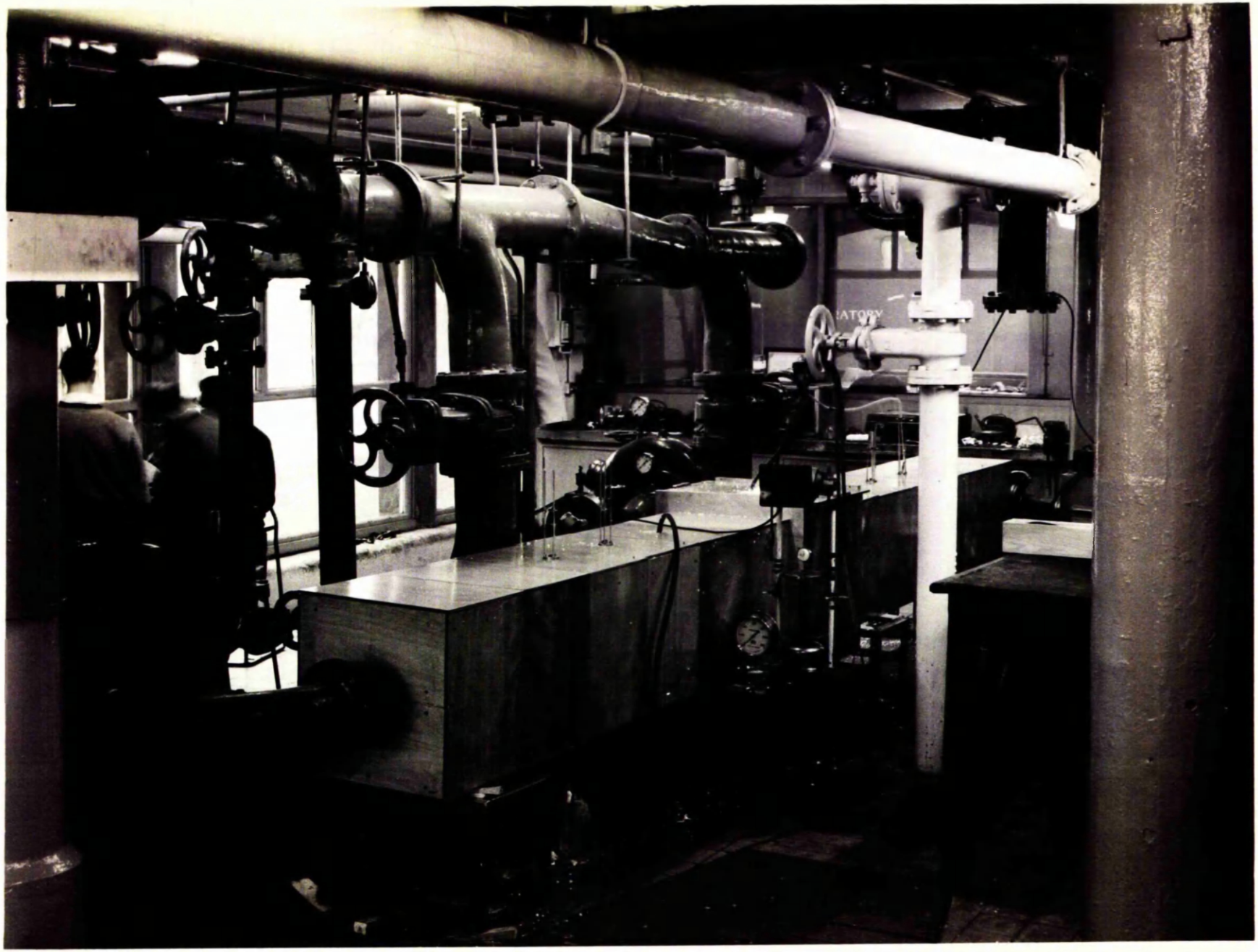


Fig. No. 8.1 The Hydraulic Throttling Calorimeter

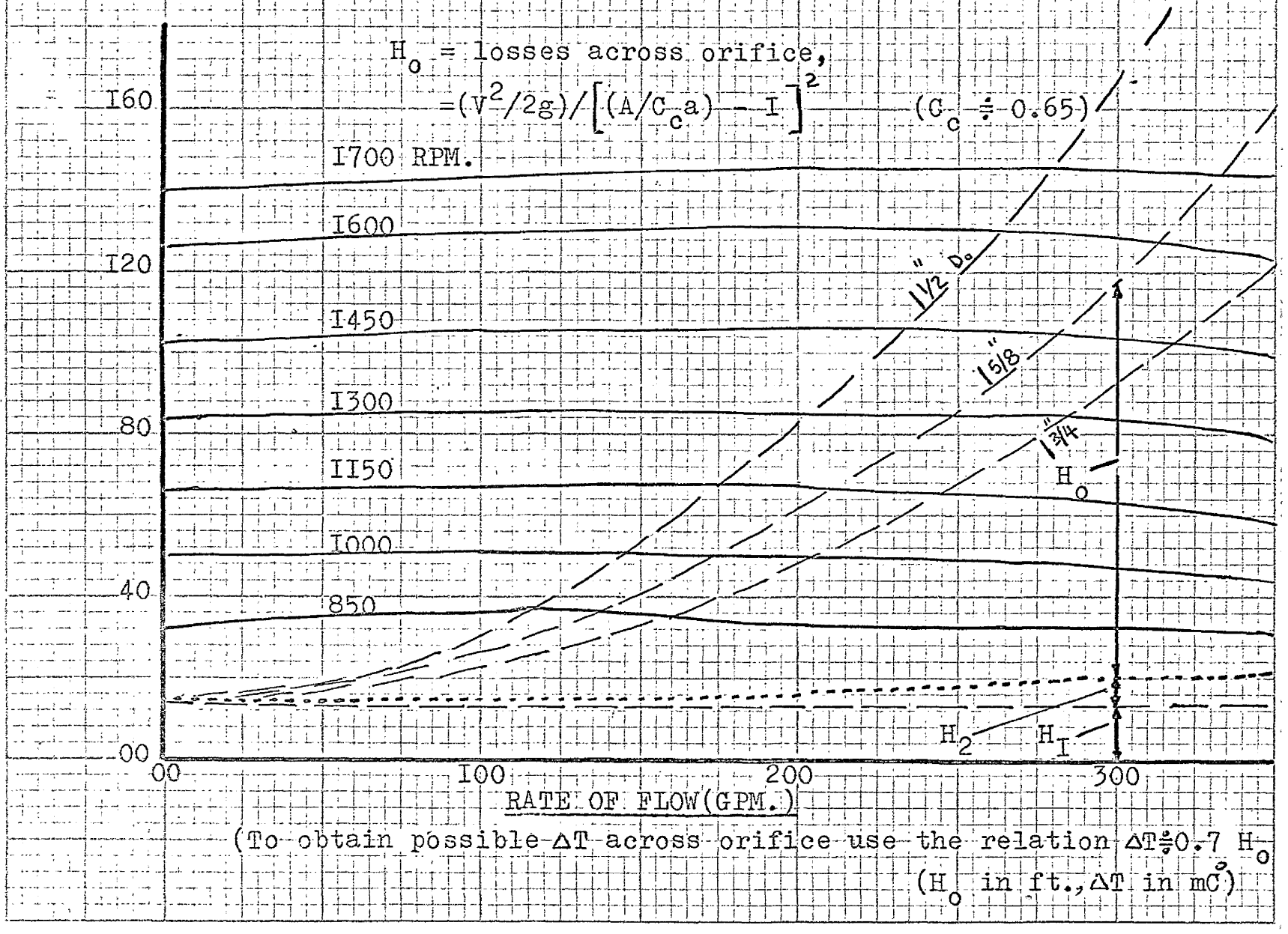
FIG.8.2 CHOICE OF SIZE OF ORIFICE

(orifices are enclosed in 5"-circuit of 4/3"pump)

H_1 = static head. H_2 = estimated overall losses besides those across orifice.

H_o = losses across orifice,

$= (V^2/2g) / \left[(A/C_c a) - 1 \right]^2$ $(C_c \doteqdot 0.65)$



(To obtain possible ΔT across orifice use the relation $\Delta T \doteqdot 0.7\ H_o$ (H_o in ft., ΔT in mC°)

TABLE NO. (8.1). (ADAPTED FROM N.E.L. REPORT NO. 130 by Foord, Langlands and Elagib.)

Test No.	A.C. Bridge Measurements		ΔT *	Throttling Calorimeter Measurements				ΔT
	(a) Thermometer x as monitor n	(b) Thermometer y as monitor n	by the pair ratio method Chapter 7 and Appendix (7.2)	P_1 lb/in^2	P_2 lb/in^2	Flow galls/min	Inlet Temp. C^o	(from Eqn. 8.1) m °C
1	0.483741	0.483771	16.2	11.41	0.79	123	15.6	16.7
2	0.483731	0.483780	25.6	16.96	0.74	167	14.9	26.3
3	0.483717	0.483794	40.1	26.14	0.74	211	14.8	41.1
4	0.483709	0.483802	49.6	32.18	0.74	237	15.1	49.8
5	0.483699	0.483813	61.5	39.77	0.74	263	15.3	62.1
6	0.483687	0.483825	73.8	47.58	0.74	287	15.3	73.9
7	0.483679	0.483833	83.2	53.56	0.74	305	15.5	82.5

Comparison of determinations of ΔT by

(a) electrical measurements with a.c. bridge and
(b) mechanical measurements on throttling calorimeter.

* ΔT is an average of the values obtained by (a) and (b) above.

TABLE (8.2)

CALIBRATION OF PLATINUM-IN-PYREX RESISTANCE THERMOMETERS BY THE A.C. BRIDGE AND THE HYDRAULIC THROTTLING CALORIMETER

Thermometer No.	R at 11°C	$(1/A_m)$			Average $(1/A_m)$	Average (A_m)
		Test No. 1	Test No. 2	Test No. 3		
1003	99.6288	265.5	261.3	260.0	262.3	0.003812
1004	99.6388	267.5	260.2	264.2 259.8	262.2	0.003814
1005	99.9832	267.5	260.7	258.4	261.2	0.003828
1006	99.4992	263.7	259		261.4	0.003826
1007	100.0000	261.1	261.7	262.3	261.7	0.003821

Tests are conducted with ΔT of the order of 45-65 m°C.

The values of the resistances R at 11°C are quoted as percentage of the value of R of thermometer No. 1007. The percentage is determined by the pair ratio measurement. These thermometers are subsequently used successfully for efficiency tests No. 7, 8, 9 and 10 in Chapter 11.

Estimated accuracy of calibration: $\pm$ 0.9 %.

A HYDRAULIC BRIDGE FOR MEASUREMENT OF SMALL TEMPERATURE DIFFERENCE

9.1 Introductory Description

The hydraulic bridge* is an instrument for the accurate measurement of small static temperature differences of the order of several thousandths of a degree, such as those occurring in flowing fluid systems.

The bridge is extremely simple to make and to use. It consists of four resistance thermometers (or more) arranged in the form of a Wheatsone bridge, Fig. (9.1). The outer two thermometers, (1) and (2), are the temperature probes and are placed at the measuring points. The inner two thermometers, (3) and (4) are used for balancing the bridge, and are placed in a pipe of flowing water, one on each side of a throttle valve. Instead of the conventional method of balancing by knob-operated resistances, the bridge is entirely balanced by manipulating the throttle-valve and hence heating or cooling the thermometer downstream of it. By noting the pressure difference across the valve, the temperature difference

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* Elagib, J. of Scientific Instruments, (Ref. 17)

sensed by the outer thermometers can be computed or obtained from the calibration chart of the bridge.

A simple method of calibrating the bridge is explained and results of proving tests are shown.

9.2. Theory

From the principles of resistance thermometery it is proved that at balance of the bridge,

$$\Delta T = m \Delta T' - C, \qquad \text{Appendix (9.1)}$$

where, ΔT and $\Delta T'$ are the temperature differences across the machine and the valve (respectively) in Fig. (9.1)

$$m = \frac{(R_o)_1 (R_o)_4 (1 + a_1\bar{T}_1) (1 + a_4\bar{T}_3) (\frac{1}{a_2} + \bar{T}_1)}{(R_o)_2 (R_o)_3 (1 + a_2\bar{T}_1) (1 + a_3\bar{T}_3) (\frac{1}{a_4} + \bar{T}_3)} \qquad (9.2)$$

and

$$C = \left[1 - \frac{(R_o)_1 (R_o)_4 (1 + a_1\bar{T}_1) (1 + a_4\bar{T}_3)}{(R_o)_2 (R_o)_3 (1 + a_2\bar{T}_1) (1 + a_3\bar{T}_3)}\right]\left[\frac{1}{a_2} + \bar{T}_1\right] \qquad (9.3)$$

The term $b\bar{T}^2$ in the basic equation of platinum resistance thermometers* $R_T = R_o (1 + a\bar{T} + b\bar{T}^2)$, (9.4) is neglected because b is small compared with a (-5.86×10^{-7} compared with 3.98×10^{-3} respectively) and has insignificant effect on the accuracy of the results. Appendix (9.1)

* $\bar{T}$ in the equation of resistance thermometers is in degrees Celsius.

The subscripts 1, 2, 3 and 4 refer to the thermometers placed (or the measurements made) at the respective points in Fig. (9.1).

From the structure of equation (9.2) for m, and if each pair of the thermometers used have nearly equal 'a' values, then 'm' can be considered as constant. The same argument applies to the first bracket in the equation for C. In the second bracket $\frac{1}{a_2}$ for platinum is about 250, hence a variation of 2.5 C^{o} in the temperature of the fluid causes only 1% variation in C. Thus C can be assumed constant for the period of one series of tests (unless an exceptional variation of fluid temperature is noted) and equation (9.1) may be considered as linear. The calibration curve in Fig. (9.2) confirms this deduction. A simple method for the experimental determination of m and C is described in Section (9.3).

By application of equation (8.1) the temperature difference $(\Delta T)'$ across the throttle valve in Fig. (9.1) may be obtained in terms of the pressure difference, $\Delta P'$.

$$\Delta T' = \frac{(A_{T_1})'}{(C_{P_2})'} \times |\Delta P'| \qquad (9.5)$$

$(A_T)'$ and $(C_P)'$ are values of A_T and C_P for the pressure and temperature conditions at the throttling valve. Substituting for $\Delta T'$ in equation (9.1),

$$\Delta T = m \times \frac{(A_{T_1})'}{(C_{P_2})'} \times |\Delta P'| - C \qquad (9.6)$$

9.3 Calibration

To determine the values of 'm' and 'C' in equation (9.6) the bridge is calibrated using another auxiliary throttling valve in place of the machine in Fig. (9.1). By this throttling valve exact temperature difference can be created, against which the bridge may be calibrated. (Refer to Fig. 9.2 for a typical calibration chart). The values of these exact temperature differences are determined by equation (9.5)

An integral bridge unit may be made consisting of two throttling valves fitted in two parallel pipes in the same insulation box (Fig. 9.1). Thus this bridge unit will be 'self-calibrating'.

In Section 9.2, it has been mentioned that the 'a' coefficients of each pair of the thermometers should be nearly equal so that 'm' and 'C' are constant.

This is very essential for the stability of the bridge and for its independence of the absolute temperature of the water, which may be quite variable.

A simple mthod of 'matching' the thermometers, i.e. selecting pairs with equal or nearly equal 'a' coefficients - if these coefficients are not known - is to connect four randomly selected thermometers as for calibration above, (i.e. thermometers (1) and (2) are placed across the auxiliary throttle-valve). The bridge is then balanced by manipulating either of the two valves. If the selected thermometers are matched, the balance of the bridge will be maintained for a long time irrespective of the variation of the absolute temperature of the water, otherwise new pairing or new thermometers have to be used until a stable quadruple setting is obtained.

For the daily checking of the value of 'C', the outer thermometers (1) and (2) may be placed at the same pocket or adjacent pockets in the outer hydraulic circuit so that $\Delta T = 0$. Thus from equation (9.6).

$$C = m \frac{(A_T)'}{(C_p)'} \times |\Delta P'| \qquad (9.7)$$

To obtain 'C' in this way, i.e. from one pressure reading, the thermometers must be so arranged that 'C' is always positive, hence it can be balanced by a positive value of $\Delta P'$. The thermometers selected must also have equal or nearly equal values of R_o, or the pairs must have as nearly identical ratios of R_o as possible. This will give a small value of C and no pressure is wasted in initial balancing, e.g. an inequality of 0.01 Ω in one of the 100 Ω thermometers used in the bridge will require to be balanced by about 38 ft. of water (equivalent to about 25 mc^o of the value of C).

9.4 Elimination of Lead Errors

The effect of lead resistance is eliminated by using leads having the same resistance ratio as that of the thermometers e.g.

$$\frac{R_1}{R_2} = r = \frac{l_1}{l_2} \quad \therefore \quad \frac{R_1}{R_2} = \frac{R_1 + l_1}{R_2 + l_2} \qquad (9.8)$$

In the case of this bridge 'r' is approximately equal to unity (to within 0.01%), hence leads of equal resistance are used. 4-core cables are chosen. Each

cable contains two pairs of leads belonging, one to an inlet and one to an outlet thermometer (or one to an outer and one to an inner thermometer). Thus the leads are not only equal but follow each other and hence are at the same ambient temperature throughout their length. Single-wire leads are used to avoid the risk of broken strands.

If the thermometers are not exactly equal, or if it is not possible to arrange them in pairs of equal R_o ratios, then the leads may have to be made unequal to obtain balance of the bridge at a minimum value of C, e.g. by lengthening or shortening one of the leads. This procedure is very useful, yet practically it is found not to incur any significant error even though the relation in equation (9.8) above is slightly upset. This is due to the fact that any error is included in the initial estimation of C and will be carried on in about the same order of magnitude to all other readings. It is then automatically subtracted when using equation (9.1).

The error due to lead inequality may be estimated by the following approximate equation:-

$$\delta(\triangle T) = \delta\frac{1}{R} \alpha T_R \qquad (9.9)$$

Thus if leads having an inequality of ± 0.01 Ω resistance are used in a bridge of 4 x 100 Ω circuit, then a change of 10 C° in the room temperature, from the time of taking the initial reading of C, will only create ± one m C° error in the value of Δ T. However it must be remembered that the permissible limit of the lead inequality is not determined only by this error, but by its effect on the stability and steadiness of the galvanometer spot or pointer. The bridge used in the tests which are described in Chapter 11 had lead resistances of 0.28 per thermometer.

9.5 Performance

The apparatus erected in the Hydraulic laboratory at the University of Glasgow was made of 1½" Piping. Four Commercial 100 Ω Platinum-in-pyrex resistance thermometers with equal 'a' values were selected. They were inserted in ⅝" water-filled copper pockets which were fitted into the pipe perpendicular to the flow. The pipe was embedded in an 8 ft long, 2' x 2' wooden box filled with a thermal insulating material, 'Polyzote.' (It was later found, however, that heat transfer has little effect because the water used was nearly at the room temperature).

(Later another bridge was constructed with 1" dia. pipes, and with the thermometers fitted parallel to the flow. The wooden box was 4½ ft. long and 10" x 10" in cross section. This bridge was used for some of the tests described in Chapters (11) and (12).)

Water was supplied to the throttle-valve through a pipe branched off the main outer hydraulic circuit (Fig. 9.1.) Thus all the thermometers were basically at about the same temperature. The pressure difference across the valve was measured by a simple mercury manometer.

A stabilised d.c. supply was used and a current of 1mA was maintained through the thermometers. The galvanometer used was fitted with a photo-cell amplifier. The effect of stray thermal e.m.f. was mullified by the usual method of current reversing. However since the balancing of the bridge was not attained by manipulating of a slide-wire or knob-contacts, the forward and reverse readings were found almost identical indicating that these e.m.f.s. were of insignificant magnitude.

As a guide to the pressure requirement of the bridge: The measurement of a temperature

difference of 0.100°C requires a head of about 150 ft. of water at the bridge. This figure is obtained by using equation (9.6) for water at room temperature and assuming that C = o and m = 1.

In this case the temperature measuring capacity of the bridge is 1 : 1 i.e. the temperature difference across the valve is of the same order as that to be measured. Methods proposed for raising the capacity of the bridge will be discussed in Section (9.6).

The rate of response of the bridge to the valve manipulation is dependant on the rate of discharge through the valve, and on the heat capacity and conductivity of the thermometer pockets. It was found that for a discharge of 50 g.p.m. the bridge responded to the manipulation of the valve in about 10 to 20 seconds. By suitable amplification of the signal to the galvanometer, the bridge may discriminate or sense a change of about 0.3 mC° (one millidegree, mC° = 0.001C°). Theoretically the sensitivity is comparable with that of a conventional bridge consisting of resistance boxes with an infinite number of decades, but practically it is limited by the pressure fluctuations in the hydraulic system. The ultimate accuracy is dependant on the stability of the circuit and the

resistance thermometers. If this stability can be maintained an accuracy of that order (0.3 mC^{o}) may be achieved. Table (9.1) shows the results of a typical proving test.

High rate of discharge through the valve increases the rate of response of the bridge and minimises the effect of heat transfer but this has to be weighed against the economy of running the bridge. Also it is found that thermometer pockets of low heat capacity are better for quick response, but they have a poor damping effect and are less suitable for the fluctuating conditions of hydraulic flow. For this reason it was found more satisfactory to put the thermometers in copper pockets filled with water than to insert them 'naked' into the flow.

9.6 Review

It can be seen from equation (9.6) that the temperature measuring capacity of the bridge is limited by the pressure difference $\Delta P'$ available at the throttle-valve, and the ratio $\frac{(A_T)}{(C_P)}$ of the fluid used for the valve circuit($A_T/C_P = -(\frac{\partial T}{\partial P})_h$ where $(\frac{\partial T}{\partial P})_h$ is the Joule-Thomson coefficient).

The pressure difference may be increased by using a booster pump to supplement the Pressure derived from the main line, or the throttle-valve may be supplied from a separate high pressure source. However these arrangements tend to reduce the economical advantage of the bridge and are not always recommended.

The value of $^{A}T/C_P$ for water at $4^{o}C$ is about 1.67×10^{-3} $C^{o}/lb.in^{-2}$, or 7×10^{-4} C^{o} per foot head of water. Amongst fluids, water has numerically the lowest ratio $^{A}T/C_P$. It provides the least temperature/pressure measuring capacity if used for the valve circuit, but it has many obvious advantages over other fluids e.g. oils or steam. In many cases, however, it will be found preferable to use the same fluid as that used in the main circuit; then at least about 1:1 temperature measuring capacity is obtained.

Equation (9.6) also shows that the temperature measuring capacity of the bridge is dependant on the values of C and m. The highest capacity attainable with any arrangement of the bridge is when C is minimum, and little or no pressure will be wasted in initially balancing the bridge. To minimise the value of C, the thermometers may be so chosen and/or so arranged that $\frac{(R_o)_4}{(R_o)_3}_{T_1}$, is equal or slightly

greater than $\frac{(R_o)_2}{(R_o)_1 T_1}$. In the latter case however C will be negative and it will not be possible to find its value by one reading as described in para.9.4. Instead it will be necessary to take two readings, using the auxiliary throttle-valve, from which the value of C can be extrapolated, or alternatively thermometers (3) and (4) are interchanged so that thermometer (4) is in the colder pocket.

A very useful method of increasing manifold the temperature measuring capacity of the bridge, or its sensitivity is to change the values of m by using the thermometer arrangements, shown in Figs. (9.3) and (9.4) and analysed in Appendix 9.2. The accuracy attainable when such arrangements are used needs to be investigated.

Other research work envisaged is to investigate the use of thermocouples instead of resistance thermometers. This necessitates the use of a different type of circuit than the Wheatstone bridge circuit which is used hitherto.

9.7 Conclusions

The hydraulic bridge has unique advantages.

i) The temperature measurements are reduced to simple pressure measurements, and the manipulation of resistance decade-knobs is replaced by fine and smooth hydraulic heating of the thermometers.

ii) All the main resistances are at a controlled basic temperature.

iii) It is 'self-calibrating.'

iv) It provides a simple and cheap means for precise measurement of small temperature difference.

The bridge is useful in many scientific and engineering fields where measurement of small temperature difference is necessary. As verified in Para. (C) of this thesis, a promising application is its use for the determination of the efficiency of hydraulic machines by the thermodynamic methods where the difficulty of measuring small temperature difference is the main handicap. In such hydraulic installations a pressurised water supply for the hydraulic bridge is readily obtainable.

APPENDIX (9.1)

THE EQUATION OF THE HYDRAULIC BRIDGE

The basic equation for platinum resistance thermometers is

$$R_T = R_o (1 + a\bar{T} + b\bar{T}^2). \qquad (A.1.0)$$

By Taylor series

$$\Delta R_T = R_o (1 + 2b\bar{T})\Delta T + R_o (2b) \frac{(\Delta T)^2}{2}$$

$$= R_o (a + 2b\bar{T} + b\Delta\bar{T}) \Delta T \qquad (A.1.1)$$

Assuming that ΔR_T is the difference between the values of resistance of thermometer No. 2 at T_1 and T_2 (Fig. 9.1)

then

$$(R_2)_{T_2} - (R_2)_{T_1} = (R_2)_o (a_2 + 2b_2\bar{T}_1 + b_2\Delta T)\Delta T \qquad (A.1.2)$$

where

$$\Delta T = T_2 - T_1$$

Dividing equation (A.1.2) by

$$(R_2)_{T_1} = (R_2)_o (1 + a_2\bar{T}_1 + b_2\bar{T}_1^2)$$

The quotient obtained is

$$\frac{(R_2)_{T_2}}{(R_2)_{T_1}} - 1 = \frac{(a_2) + 2b_2\bar{T}_1 + b_2\Delta\bar{T})}{(1 + a_2\bar{T}_1 + b_2\bar{T}_1^2)} \quad \Delta T \qquad (A.1.3)$$

As the constant b_2 and ΔT are both very small ($b = -5.86 \times 10^{-7}$) the term $b_2\Delta T$ may be neglected.

Recasting equation A.1.3,

$$\frac{(R_2)_{T_2}}{(R_2)_{T_1}} = 1 + \frac{(a_2 + 2b_2\bar{T}_1)\,\Delta\,\bar{T}}{(1 + a_2\bar{T}_1 + b_2\bar{T}_1^{\,2})}$$

$$= 1 + M\,\Delta\,T \qquad \text{(A.I.4)}$$

similarly

$$\frac{(R_4)_{T_4}}{(R_4)_{T_3}} = 1 + \frac{(a_4 + 2b_4\bar{T}_3)\;\; X\;\Delta\,T'}{(1 + a_4\,\bar{T}_3 + b_4\bar{T}_3'^{\,2})}$$

$$= 1 + M'\,\Delta\,T' \qquad \text{(A.1.5)}$$

where

$$M = \frac{(a_2 + 2b_2\,\bar{T}_1)}{(1 + a_2\,\bar{T}_1 + b_2\bar{T}_1^{\,2})}$$

and

$$M' = \frac{(a_4 + 2b_4\bar{T}_3)}{(1 + a_4\bar{T}_3 + b_4\bar{T}_3^{\,2})} \qquad \text{(A.1.6)}$$

ΔT and $\Delta T'$ are the temperature differences across the machine and the throttle valve respectively.

When the bridge is balanced,

$$\frac{(R_2)_{T_2}}{(R_1)_{T_1}} = \frac{(R_4)_{T_4}}{(R_3)_{T_3}} \qquad \text{(A.1.7)}$$

$$\frac{(R_2)_{T_2}}{(R_2)_{T_1}} \times \frac{(R_2)_{T_1}}{(R_1)_{T_1}} = \frac{(R_4)_{T_4}}{(R_4)_{T_3}} \times \frac{(R_4)_{T_3}}{(R_3)_{T_3}} \qquad \text{(A.1.7)}$$

(According to equation 9.8 lead resistance is not effective)

Substitution for

$$\frac{(R_2)_{T_2}}{(R_2)_{T_1}} \quad \text{and} \quad \frac{(R_4)_{T_4}}{(R_4)_{T_3}} \quad \text{from (A.1.4) and (A.1.5)}$$

respectively, equation (A.1.7)

$$(1 + M \Delta T) \frac{(R_2)_{T_1}}{(R_1)_{T_1}} = (1 + M' \Delta T') \frac{(R_4)_{T_3}}{(R_3)_{T_3}}.$$

Rearranging,

$$\Delta T = \frac{(R_4)_{T_3}}{(R_3)_{T_3}} \times \frac{(R_1)_{T_1}}{(R_2)_{T_1}} \times \frac{M'}{M} \times \Delta T' + \left[\frac{(R_4)_{T_3}}{(R_3)_{T_3}} \times \frac{(R_1)_{T_1}}{(R_2)_{T_1}} - 1\right] \times \frac{1}{M} \qquad \text{(A.1.8)}$$

Substitution for the resistances from equation (A.1.0) above and for M and M' from (A.1.6) gives

$$\bar{\Delta}T = \frac{(R_1)_o\,(R_4)_o\,(1 + a_1\bar{T}_1 + b_1\bar{T}_1^2)(1 + a_4\bar{T}_3 + b_4\bar{T}_3^2)}{(R_2)_o\,(R_3)_o\,(1 + a_2\bar{T}_1 + b_2\bar{T}_1^2(1 + a_2\bar{T}_3 + b_3\bar{T}_3^2)} \times$$

$$\frac{(a_4 + 2b_4\bar{T}_3)\,(1 + a_2\bar{T}_1 + b_2\bar{T}_1^2)}{(1 + a_4\bar{T}_3 + b_4\bar{T}_3^2)(a_2 + 2b_2\bar{T}_1)} \times \Delta T' + \left[\frac{(R_1)_o\,(R_4)_o}{(R_2)_o\,(R_3)_o} \times \right.$$

$$\left.\frac{(1 + a_1T_1 + b_1\bar{T}_1^2)(1 + a_4\bar{T}_3 + b_4\bar{T}_3)}{(1 + a_2T_1 + b_2\bar{T}_1^2)(1 + a_3\bar{T}_3 + b_3\bar{T}_3^2)} - 1\right]\left[\frac{1 + a_2\bar{T}_1 + b_2\bar{T}_1^2}{a_2 + 2b_2\bar{T}_1}\right]$$

(A.1.9)

Since b is small compared with a, the terms containing b can be neglected and equation A.1.9 may be written as

$$\Delta T = \frac{(R_1)_o\,(R_4)_o\,(1 + a_1\bar{T}_1)\,(1 + a_4\bar{T}_3)\,(\frac{1}{a_2} + \bar{T}_1)}{(R_2)_o\,(R_3)_o\,(1 + a_2\bar{T}_1)\,(1 + a_3\bar{T}_3)\,(\frac{1}{a_4} + \bar{T}_3)} \times \Delta T'$$

$$+ \left[\frac{(R_1)_o\,(R_4)_o\,(1 + a_1\bar{T}_1)\,(1 + a_4\bar{T}_3)}{(R_1)_o\,(R_3)_o\,(1 + a_2\bar{T}_1)\,(1 + a_3\bar{T}_3)} - 1\right]\left[\frac{1}{a_2} + \bar{T}_1\right]$$

(A.1.10)

$$= m\,\Delta T' - C, \qquad \text{(A.1.11)}$$

where

$$m = \frac{(R_1)_o\,(R_4)_o\,(1 + a_1\bar{T}_1)\,(1 + a_4\bar{T}_3)\,(\frac{1}{a_2} + \bar{T}_1)}{(R_2)_o\,(R_3)_o\,(1 + a_2\bar{T}_1)\,(1 + a_3\bar{T}_3)\,(\frac{1}{a_4} + a_3\bar{T}_3)} \qquad \text{(A.1.12)}$$

$$C = \left[1 - \frac{(R_1)_o\,(R_4)_o\,(1 + a_1\bar{T}_1)\,(1 + a_4\bar{T}_3)}{(R_2)_o\,(R_3)_o\,(1 + a_2\bar{T}_1)\,(1 + a_3\bar{T}_3)}\right]\left[\frac{1}{a_2} + \bar{T}_1\right]$$

(A.1.13)

Equation (A.1.10) above is also applicable to thermometers made of metals other than platinum, whose value of b = o.

APPENDIX (9.2)

STEPPING UP THE MEASURING CAPACITY OF THE BRIDGE

(Refer to Fig. (9.3))

At balance of the bridge

$$\frac{(R_{2a})_{T_2} + (R_{2b})_{T_1}}{R_{T_1}} = \frac{(R_4)_{T_4}}{(R_3)_{T_3}} \qquad (A.2.0)$$

or,

$$\frac{(R_{2a})_{T_2} + (R_{2b})_{T_1}}{(R_{2a})_{T_1} + (R_{2b})_{T_1}} \times \frac{(R_{2a})_{T_1} + (R_{2b})_{T_1}}{R_{T_1}} = \frac{(R_4)_{T_4}}{(R_4)_{T_3}} \times \frac{(R_4)_{T_3}}{(R_3)_{T_3}} \qquad (A.2.1)$$

The term $\frac{(R_{2a})_{T_2} + (R_{2b})_{T_1}}{(R_{2a})_{T_1} + (R_{2b})_{T_1}}$ may be simplified as following

$$\frac{(R_{2a})_{T_2} + (R_{2b})_{T_1}}{(R_{2a})_{T_1} + (R_{2b})_{T_1}} = \frac{(R_{2a})_{T_2} - (R_{2a})_{T_1} + (R_{2b})_{T_1} + (R_{2a})_{T_1}}{(R_{2a})_{T_1} + (R_{2b})_{T_1}}$$

$$= \frac{(R_{2a})_{T_2} - (R_{2a})_{T_1}}{n\,(R_{2a})_{T_1}} + 1$$

$$= \frac{1}{n}\left[\frac{(R_{2a})_{T_2}}{(R_{2a})_{T_1}} - 1\right] + 1$$

$$= \frac{1}{n}\, M \Delta T + 1 \qquad \text{(A.2.2)}$$

(by equation A.1.4)

where, $n = \dfrac{(R_{2a})_{T_1} + (R_{2b})_{T_1}}{(R_{2a})_{T_1}}$

By substituting for $\dfrac{(R_{2a})_{T_2} + (R_{2b})_{T_1}}{(R_{2a})_{T_1} + (R_{2b})_{T_1}}$ and $\dfrac{(R_4)_4}{(R_4)_3}$ from equations A.2.2 and A.1.5 respectively and recasting, equation A.2.1. becomes

$$\Delta T = n\left(\left[\frac{(R_4)_{T_3}}{(R_3)_{T_3}} \times \frac{(R_1)_{T_1}}{(R_{2a})_{T_1} + (R_{2b})_{T_1}}\right] \times \frac{M'}{M} \Delta T' + \left[\frac{(R_4)_{T_3}}{(R_3)_{T_3}} \times \frac{(R_1)_{T_1}}{(R_{2a})_{T_1} + (R_{2b})_{T_1}} - 1\right] \times \frac{1}{M}\right) \qquad \text{(A.2.3)}$$

M & M' are defined in Appendix 9.1. Equation A.2.3 shows that to balance a temperature difference ΔT only about $\frac{1}{n}\Delta T$ is required at the bridge.

Stepping up the sensitivity of the bridge

(Refer to Fig. (9.4)).

The same analysis above is applied to thermometers (4a) and (4b) with the result,

$$\frac{(R_{4a})_{T_4} + (R_{4b})_{T_3}}{(R_{4a})_{T_3} + (R_{4b})_{T_3}} = \frac{1}{n} \times M' \times \Delta T' + 1 \qquad (A.2.4)$$

and

$$\Delta T = \frac{1}{n}\left(\left[\frac{(R_{4a})_{T_3} + (R_{4b})_{T_3}}{(R_3)_{T_3}} \times \frac{(R_1)_{T_1}}{(R_2)_{T_1}} \times \right] \frac{M'}{M} \times \Delta T' + \left[\frac{(R_{4a})_{T_3} + (R_{4b})_{T_3}}{(R_3)_{T_3}} \times \frac{(R_1)_{T_1}}{(R_2)_{T_1}} - 1 \right] \times \frac{1}{M} \right) \qquad (A.2.5)$$

In this case the sensitivity is multiplied 'n' times.

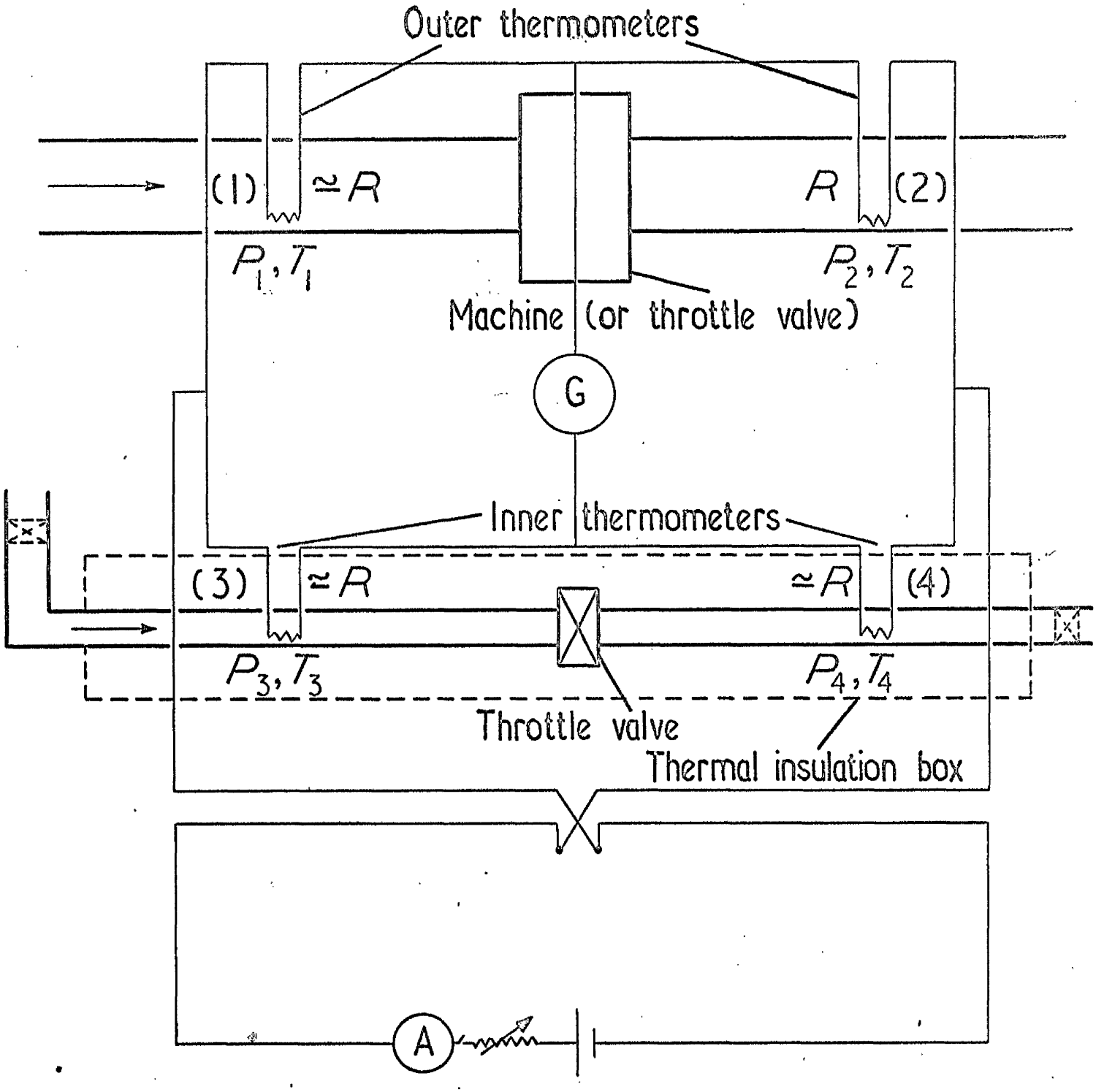

FIG. 9.1 *Schematic arrangement of the hydraulic bridge (m = 1).*

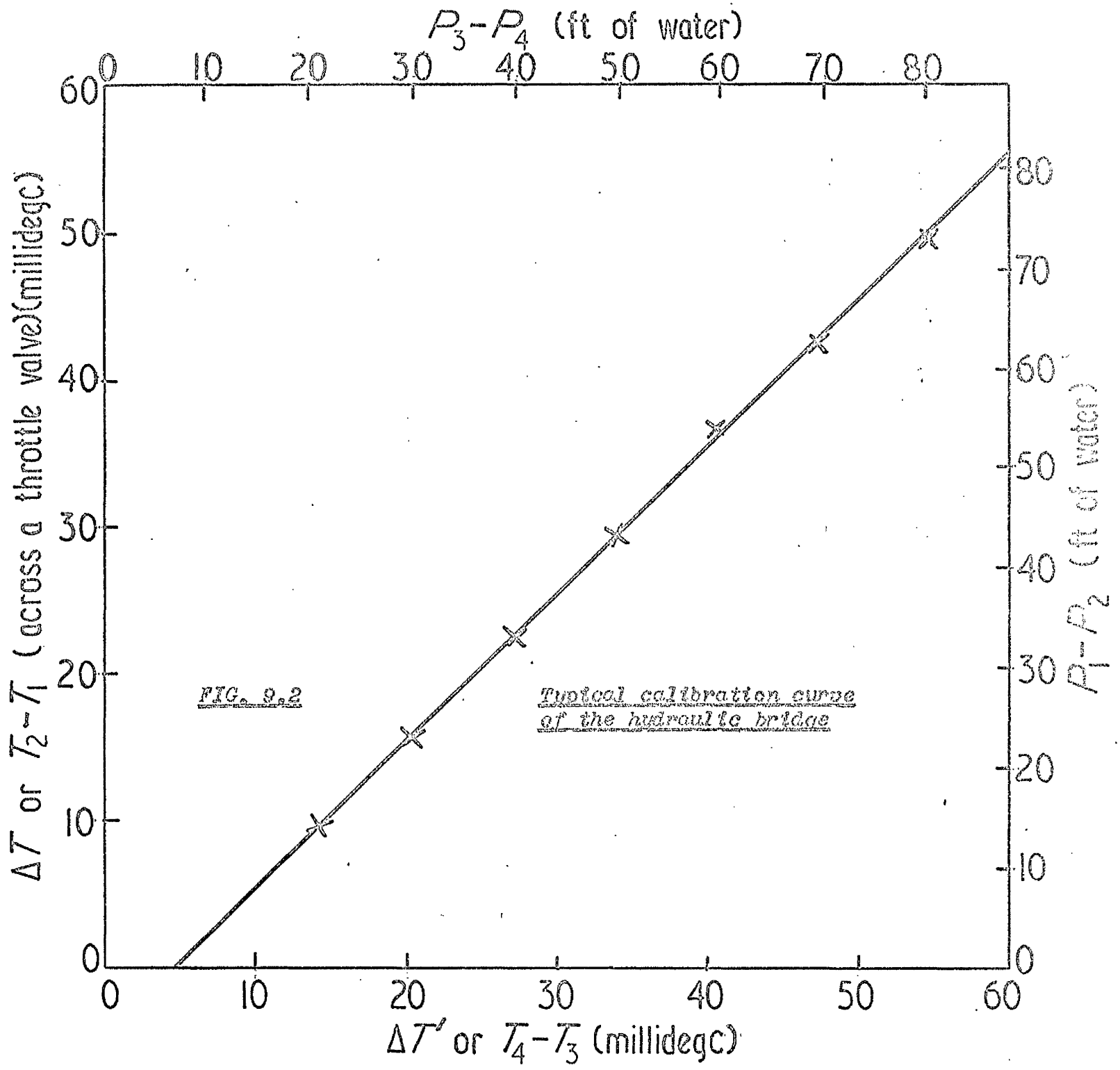

FIG. 9.2 Typical calibration curve of the hydraulic bridge

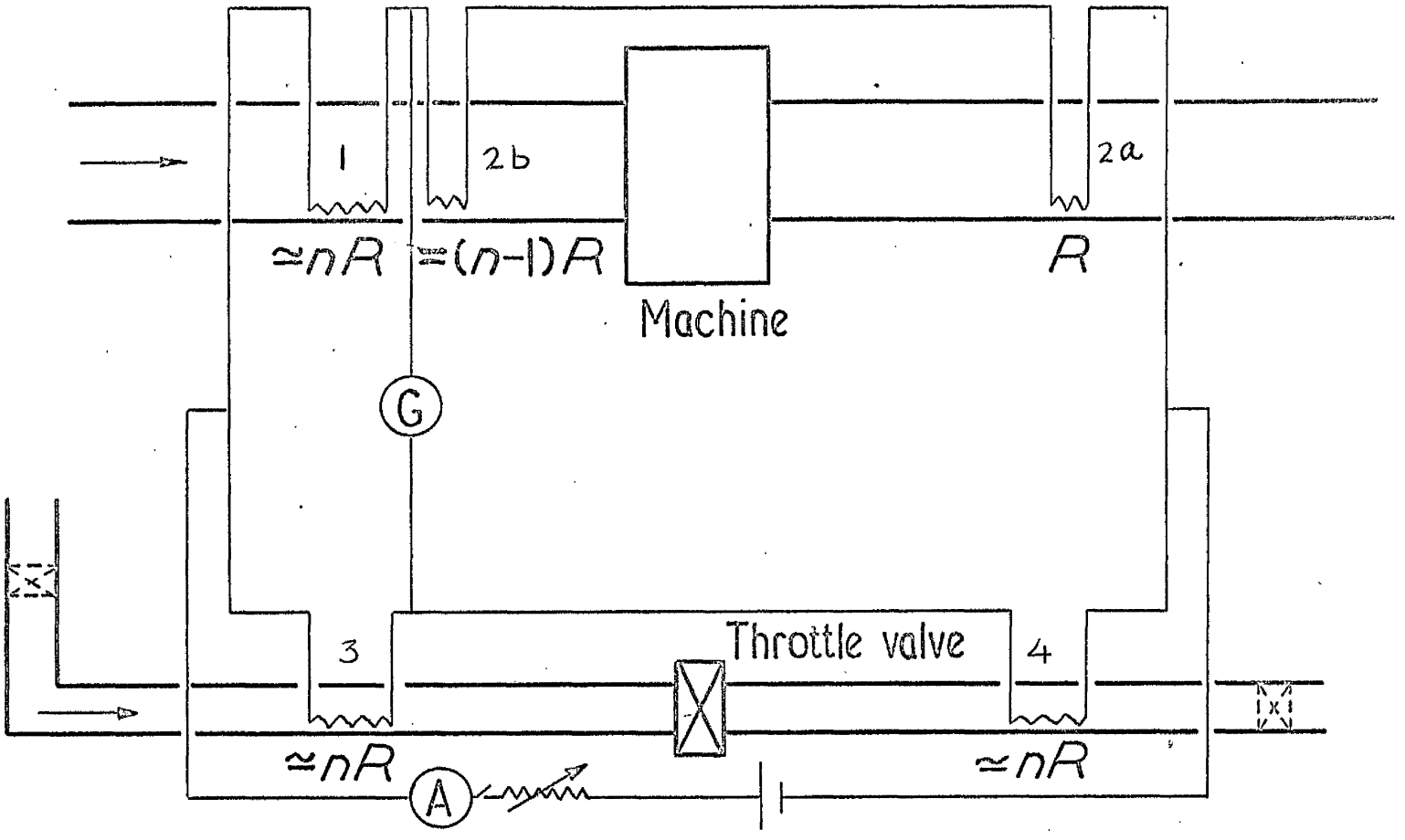

FIG. 9.3 *Arrangement for stepping up the measuring capacity of the bridge* $(m > 1)$

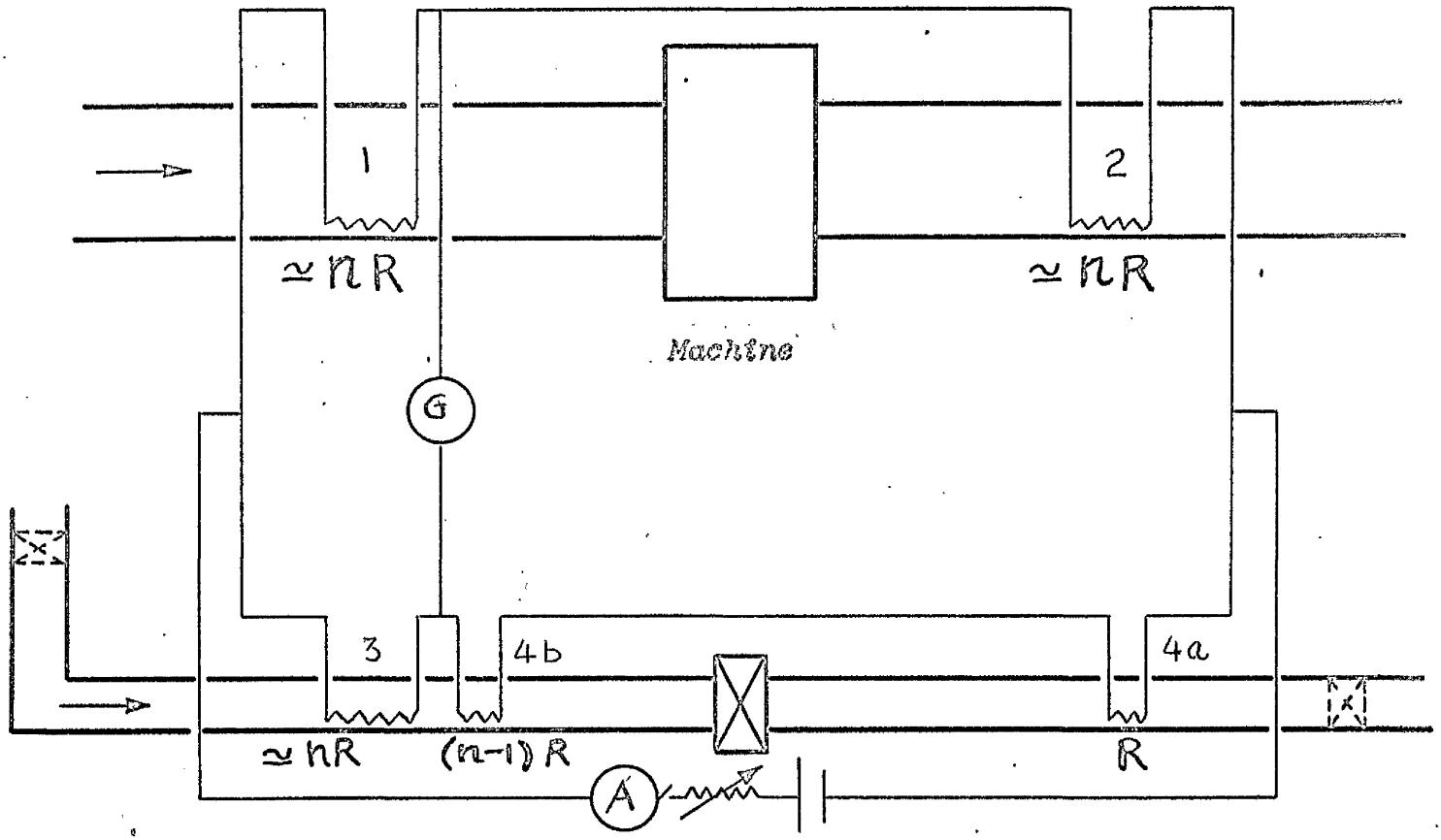

FIG. 9.4 *Arrangement for stepping up the sensitivity of the bridge* ($m < 1$).

TABLE (9.1)

MEASUREMENT OF TEMPERATURE DIFFERENCE ACROSS A THROTTLE-VALVE BY THE HYDRAULIC BRIDGE

Test No.	ΔT, calculated from Pressure measurements by Equation (9.5)	ΔT measured by the hydraulic bridge
	mC^{o}	mC^{o}
1	6.5	6.4
2	12.5	12.0
3	20.2	19.8
4	27.7	27.4
5	35.2	35.3
6	42.9	42.6

CHAPTER TEN

THE THERMOMETER - POCKET CORRECTION

10.1 Introduction

From theories of the boundary layer, it is known that the temperature, T_m, measured at a stationary adiabatic pocket inserted across a flowing fluid, is higher than the temperature, T_∞, at the free-stream. This difference or pocket error is due to two factors. Firstly the isentropic change of the temperature of the fluid associated with its acceleration and deceleration (and hence its expansion and compression) on its path round the pocket. Secondly the frictional or viscous dissipation of the kinetic energy of the fluid in the boundary layer of the pocket. The difference of temperature, $(T_m - T_\infty)$, is usually expressed as a fraction of the dynamic temperature, $(U_\infty^2/2g_uC_p)$. The ratio of the difference to the dynamic temperature is known as the recovery factor which is dependent on the thermodynamic and transport properties of the fluid and on the type of flow (Ref. 18).

This problem of stagnation and frictional heating is well studied in the field of high speed flight and heat transfer, but is rarely considered

in the normal practice of temperature measurement. In most cases of hydraulic flow the effect may amount to only several thousandths of a degree. Neverthless this may cause a significant error if neglected in a thermodynamic treatment of liquid flow. For example in the efficiency tests on the 4"/3" pump by the thermodynamic methods described in Chapter II, its neglect might have caused an error of about 6% in the measurement of the efficiency.

To determine the pocket correction, $(T_{\infty} - T_m)$, two methods are proposed. The first is a direct experimental method. The second is based on the use of a semi-empirical expression for the recovery factor which was developed during the course of this research.

10.2 Experimental method for the determination of the pocket correction

In many of the problems of fluid flow, the accurate measurement of the temperature difference is required rather than the measurement of the absolute temperature. Thus the correction which usually needs to be determined is that of the termperature difference, namely - $\delta(\Delta T)$.

$$\delta(\Delta T) = \Delta T_m - \Delta T,$$
$$= (T_{m2} - T_{m1}) - (T_2 - T_1) \qquad (10.1)$$

In case of liquid flow, if the sections of the pipe at which the pockets are located are identical, the velocities will be equal and hence

$$T_{m1} - T_1 = T_{m2} - T_2$$

No pocket correction will be needed. If the sections are of different dimensions, as is the case for the 4"/3" pump mentioned above, the velocities will be different necessitating a pocket correction. To determine this correction two pipes of 4" and 3" diameters were laid horizontally in the calorimeter box (described in Chapter 8) in place of the 5" pipe. The pipes were arranged in the same way as those fitted on to the pump, i.e. the 4" pipe was laid in the upstream side. Readings of the temperature and pressure differences across the constriction were taken for the same rates of discharge at which the pump was tested. The a.c. bridge and the pair ratio method were used for the measurement of the temperature difference. The measured temperature difference was compared with the actual difference ΔT, which was calculated by equation (4.14),

$$\Delta T = T_2 - T_1 = (1/C_p) \left[A_{T_1} (P_1 - P_2) + (V_1^2 - V_2^2) / 2g_u \right] \tag{10.2}$$

The corrections listed in Table (11.2) and used for tests 1, 2, 3, and 4 (on the $4^{n}/3^{n}$ pump) in Chapter II were obtained by the same procedure described above. Also the same procedure was applied in the case of the efficiency tests Nos. 7, 8, 9 and 10 (on the reversible pump/turbine model described in the same Chapter, (11)). However, in this case, no separate pipes were used. Instead, the pockets were calibrated 'in situ' with the impeller locked. The corrections in these two cases are quoted in this Chapter in Table (10.2).

10.3 A General Expression for the overall recovery factor of a stream-line pocket normal to the flow

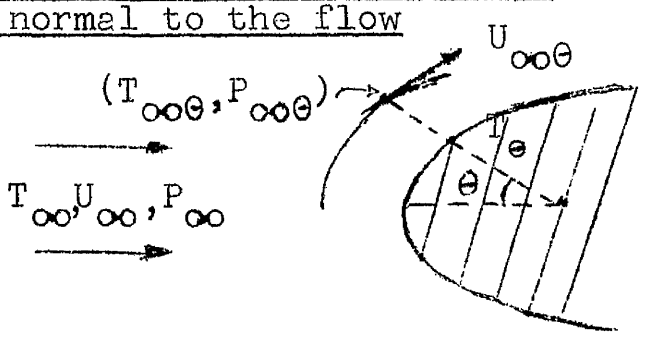

Fig. 10.0

Consider the case of a stream-line, adiabatic pocket held perpendicular to the flow. (A cross section of the pocket is shown in Fig. 10.0).

Consider first that the pocket is made of a perfectly non-conductive material so that each point on the surface of the pocket is free to assume a

temperature different from that of the adjacent points.

$$T_{\theta} = T_{\infty\theta} + r_{f\theta} \left(U^2_{\infty\theta} / 2 g_u C_p\right) \qquad (10.3)$$

r_{fo} is the local frictional recovery factor and $r_{fo} \left(U^2_{\infty\theta}/2g_uC_p\right)$ is the local temperature rise due to frictional heating or viscous dissipation of the energy of flow.

By assuming that along a streamline at the outer edge of the boundary layer, the transformation from U_{∞} to $U_{\infty\theta}$ occurs isentropically, and by the aid of equation (3.14) for the change of temperature accompanying an isentropic change of velocity,

$$T_{\infty\theta} = T_{\infty} + S_c \left(1/2g_uC_p\right)\left(U^2_{\infty} - U^2_{\infty\theta}\right) \qquad (10.4)$$

where

S_c, the stagnation coefficient =

$$(T/v)(\partial v/\partial T)_p \qquad (10.4.a)$$

(Section 3.3)

$(U^2_{\infty} - U^2_{\infty\theta})$ may be determined from the relation

$$P_{c\theta}\, U^2_{\infty} /2g_u = (U^2_{\infty} - U^2_{\infty\theta})/2g_u \qquad (10.5)$$

where $P_{c\theta}$ the pressure coefficient

$$= v\,(P_{\infty\theta} - P_{\infty})/(U^2_{\infty}/2g_u) \qquad (10.5.a)$$

Thus equation (10.4) may be re-written as,

$$T_{\infty\Theta} = T_{\infty} + S_c P_{c\Theta} (U_{\infty}^2/2g_u C_p) \qquad (10.6)$$

Subsequently equation (10.3) is recast as

$$T_{\Theta} = T_{\infty} + (1-P_{c\Theta}) r_{f\Theta} + S_c P_{c\Theta} (U_{\infty}^2/2g_u C_p) \qquad (10.7)$$

However since the thermometer-pocket is usually made of a conductive material, e.g. copper, it will assume an average temperature, T_m, which is an average of the 'point' temperatures represented by equation (10.7).

$$T_m = (1/\pi L) \int_0^L \int_0^{\pi} T_{\Theta} \, d\Theta \, dL \qquad (10.8)$$

If T_{Θ} is independent of L, e.g. if L is very long and the flow is turbulent equation (10.8) may be reduced to the simple form

$$T_m = (1/\pi) \int_0^{\pi} T_{\Theta} \, d\Theta. \qquad (10.8.a)$$

Furthermore, if the conditions of the fluid and of the flow around the pocket are approximately the same throughout the circumference of the pocket, $r_{f\Theta}$ and S_c may be assumed constant and independent of Θ. As a result the integration of equation (10.8.a) yields,

$$T_m = T_{\infty} + \left[(1-P_c) r_f + P_c S_c (U_{\infty}^2/2g_u C_p)\right] \qquad (10.9)$$

$$(r_{f\Theta 1} = r_{f\Theta 2} = \ldots = r_f)$$

$$\text{or } T_m = T_\infty + R_f \, (U_\infty^2 / 2g_u C_p) \qquad (10.9.a)$$

$$\text{where} \quad P_c = (1/\pi) \int_0^\pi P_{c\theta} \, d\theta \qquad (10.9.b)$$

$$\text{and } R_f = (1-P_c) r_f + P_c S_c \qquad (10.10)$$

= The overall recovery factor.

In equation (10.10), S_c is a property of the fluid and may be determined by equation (10.4.a) Values for water may be obtained from Fig. (10.4) P_c is dependent on the configuration of the pocket and may be determined by averaging the measured (or calculated) values of $P_{c\theta}$ as shown in section (10.5). r_f (or $r_{f\theta}$), the frictional recovery factor is also a property of the fluid and may be obtained from Eckert and Drewitz's solution for a flat plate at zero incidence to the flow, where the recovery effect is wholly frictional,

$$r_f = f(P_r) = P_r^n \quad \text{(Ref. 19)} \qquad (10.11)$$

Hence equation (10.10) may be written as,

$$R_f = (1 - P_c) P_r^n + P_c S_c \qquad (10.12)$$

The values of n for a laminar boundary layer on the flat plate and for Prandtl numbers up to 40 are shown in Fig. (10.1) ($n \doteq \frac{1}{2}$). These are quoted from Eckert and Drewitz's data. For turbulent boundary layers on a flat plate Ackermann (Ref. 20) suggested

that for $0.5 < P_r < 2$ n may be approximated by 1/3.

The assumption that the local frictional recovery factor $r_{f\theta}$ is constant around the circumference of the stream-line pocket is based on the results of an interesting test conducted by Eckert and Weise (Ref. 21), in which they measured the local recovery factor at intervals round the periphery of a non-conductive circular cylindrical pocket held normal to air flow. Their results show that at $\theta = o$ the recovery factor is constant throughout the region where the fluid is attached to the pocket, i.e. before the point of separation. Their results also show that its value is identical with that for laminar boundary layer on a flat plate ($r_f = P_r^{\frac{1}{2}}$). At $\theta = o$, they found that the ratio of the temperature rise to the dynamic temperature is equal to unity confirming that the rise is due to stagnation.

The validity of equations (10.7), (10.9), (10.10) and (10.12) may be verified by applying them to the following cases.

(i) Flat Plate (zero-incidence to flow)

$P_c = o$, therefore by equation (10.10), $R_f = r_f$.

This is what may be expected since the recovery factor of a flat plate is due to friction only.

(ii) The leading edge of a pocket

$P_{c\Theta} = 1$, therefore by equation (10.7)

$$T_\Theta - T_\infty = S_c \, (U_\infty^2 / 2g_u C_p).$$

This equation verifies the fact that the temperature difference at $\Theta = o$ is due to stagnation only.

(iii) Inviscid flow round a cylinder

$\mu = o$, $P_r = o$ and $P_c^* = -1$.

Thus by equation (10.12), $R_f = - S_c$

and by equation (10.9) $T_m = T_\infty - S_c \, (U_\infty^2 / 2g_u C_p)$

This shows that the pocket temperature is influenced only by the phenomenon of stagnation.

10.4 Adaptation of Equations (10.9.a) and (10.12) to practical conditions

For practical convenience the overall recovery factor, R_f, will be based on the average velocity, (Q/A), instead of U_∞. Thus equation (10.9.a) is re-written as

$$T_m = T_\infty + R_f \, (V^2 / 2g_u C_p) \qquad (10.13)$$

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* This is evolved by using the relation $U_{\infty\Theta} = 2 \, U_\infty \sin\Theta$

In all the experiments carried out in this research the pipes used were of small size and the pockets were so fitted that they almost spanned the whole diameter of the pipe. For this condition the relation between the overall recovery factor based on V and that based on U_{∞} is as following.

$$R_f \text{ based on } V = K_v \times R_f \text{ based on } U_{\infty} \qquad (10.14)$$

K_v is a coefficient dependent on the Reynolds number.

$$K_v = (1/L) \int_0^L (U/V)^2 \, dL \qquad (10.15)$$

A table of K_v is shown in Appendix (10.1). The table is based on the assumption that L = D. (In case of large pipes the pocket is usually located at the position where V = U and K_v = 1 (Refs. 10 and 12)).

Equations (10.9), (10.10) and (10.12) were derived for a streamline pocket with an infinite span or infinite area of flow around it, and with the assumption that r_{fe} is constant all around the pocket. However in this research as in most cases of general practice the pockets used are of circular cylindrical form and this constancy of r_f is likely to be upset by the phenomenon of separation. Thus if equation 10.12 is applied to these pockets the index 'n' may be different from that of a flat plate. This difference

will be aggravated by the proximity of the enclosing pipe and its boundary layer if the pipes used are of small size. Consequently it may be expected that n will be closely dependent on the range of the Reynold number and the ratio (d/D).

To explore the effects of these practical conditions the following tests were conducted.

10.5 Experimental Determination of P_c, the average coefficient of pressure distribution

Ideally the value of P_c must be determined for a number of sections covering the whole span of the pocket. However since the recovery factor is to be based on the average velocity (Q/A), it is appropriate (and convenient) to use the point where the absolute velocity is equal to the average velocity.

A 1"/64 D. hole was drilled in the pocket so that it lies at 0.32 D from the centre of the pipe. This figure is slightly lower than the usual figure of 3/8 D quoted for high Reynold numbers (Appendix 10.1), and it is appropriate for the range of Reynold numbers of the flow used in the experiments described in the following sections (Ref. 12).

The pocket was then fitted in place and connected to one limb of a mercury manometer. The other limb of the manometer was connected to the static pressure hole in the wall of the pipe. The pocket was rotated about its axis until the maximum positive pressure difference ($P_\theta - P_\infty$) was shown on the manometer, indicating that the hole was facing the flow ($\theta = o$). $U_\infty|_{\theta=o}$ was determined from the relation

$$U_\infty^2/2g_u = v\,(P_o - P_\infty)$$

The pocket was rotated in increments of 15^o until $\theta = 180^o$. The values of P_{co} for different pockets were plotted in fig. (10.2). P_c the average value of $\left[P_{c\theta}\right]_o^{180^o}$ was obtained by the usual method of averaging the area under the curve of $P_{c\theta}$. (Table 10.1)

10.6 Experimental Determination of R_f

The procedure adopted was the same as that described in section (10.2)(2) for the determination of the pocket correction. The two pipes used were of diameters $D_1 = 4''$ and $D_2 = 1\frac{1}{2}''$. The flow was supplied by a $3''$ centrifugal pump which was rated at 300 gpm at 15 feet head. The velocity head $V_1^2/2g$ in the large pipe, D_1, was of the order of 2% of that in the smaller

pipe. Hence the temperature error, $(T_{m1} - T_1)$ may be neglected and equation (10.1) may be re-written as

$$\Delta T_m - \Delta T = (T_{m2} - T_2)$$

$$= R_f \, (V_2^2 / 2g_u C_p) \qquad (10.17)$$

(by equation 10.13)

The use of binary pipes or pockets was preferred, because differential measurements of the temperature of a flowing fluid are more accurate than the measurement of the absolute temperature which would have been necessary if a single pocket was used.

To investigate the effect of the ratio (d/D) different sizes of cylindrical pockets were used ($\frac{1}{4}$", $\frac{3}{8}$", $\frac{1}{2}$" and 5"/8 D.). The experiments with the 4" and $1\frac{1}{2}$" pipes were carried out in the University of Glasgow. Later, experiments with large pipes, 20" and 8" were conducted in the National Engineering Laboratory at East Kilbride. The results of these tests are shown in Tables (10.1) and (10.2).

In all these experiments ΔT_m was measured by the a.c. bridge and using the pair ratio method. The rate of flow was determined by weighing. ΔT and R_f were calculated by equations (10.2) and (10.17) respectively.

10.7 The 'Pluracol' experiments

These experiments were designed to verify the dependence of the recovery factor on the Prandtl number. Water was mixed with different proportions of 'Pluracol' thus providing solutions with different Prandtl numbers.

Pluracol is a high molecular weight high viscosity liquid of the chemical group 'Polyoxyalkylene Polyol.'* It is a powerful agent for viscosity control; yet it has very little effect on the thernomdynamic properties of water. A 3% (by weight) pluracol/water solution has a viscosity of about three times that of water. The manufacturers claim that it has many unique properties which make it very suitable for experimental studies involving the variation of viscosity. Some of these properties are quoted below from manufacturers' data.

(i) Soluble in water in all proportions below 77°C.

(ii) Non volatile, non hygroscopic, non-corrosive, thermally stable below 232°C and fire resistent with a flash point of 265°C.

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* A product of Wyandotte Chemicals International inc.

(iii) A water/pluracol solution exhibits Newtonian properties even at high concentrations, (e.g. 30%).

(iv) As mentioned above, pluracol has little effect on the thermodynamic properties of water (S.G. of Pluracol at 15°C is 1.014).

Values of the prandtl number, the kinematic viscosity, and the thermal conductivity for the different pluracol solutions used are shown in Figures (10.5, 2 & 3). These curves are based on results of tests on the actual solutions used in the experiments. The values of the specific volume, the coefficient of pressure enthalpy, A_T, and the stagnation coefficient, S_c, for these solutions (concentration $< 4\%$) are practically the same as those for water.

The mixing of pluracol and water is done by pouring the appropriate amount of pluracol in the discharge tank of the pump mentioned in the foregoing section and directing the jet of water from the discharge pipe on to it. The Pluracol is thus gradually licked away from the Bottom of the tank and thorough mixing was attained.

The values of R_f for the different pluracol solutions were determined by the same method described in the preceding section, and using the same 4" and $1\frac{1}{2}$" pipe combination. For each solution the experiment was repeated with pockets of different sizes i.e. ($\frac{1}{4}$", $\frac{3}{8}$", $\frac{1}{2}$" and 5/8" D.) The results of these tests are shown in Tables (10.1). Details of an analysis of the results based on equation (10.12) are shown in Table (10.1).

10,9 Discussion of the Results

The results shown in Table (10.1) verify the dependence of the overall, recovery factor on (a) the Prandtl number of the fluid, (b) the average coefficient of pressure distribution, (c) the Reynolds number of the flow and (d) the ratio d/D.

The relation of the recovery factor with the Prandtl number was inconsistent when the 4% Pluracol solution was used. It needs to be investigated whether this is due to an abnormality in the behaviour of the Pluracol or whether there exists a critical limit for the value of R_f, typical of a certain pocket lay-out. It was observed, however, that the solution was

becoming very frothy and it was not thought worthwhile experimenting with higher concentrations.

Although the results obtained by carrying out the integration of $P_{c\theta}$ from 0^{o} up to 180^{o}, were fairly consistent, the validity of carrying the integration beyond the point of separation is still to be confirmed, probably by a repetition of Eckert and Weise's experiments on a non-conductive pocket (Section 10.3) and correlating the results with the overall recovery factor obtained for a conductive replica of the pocket.

It can be seen from Table (10.1) that there is a significant difference between the results obtained with large pipes (small d/D ratios $\leq$ .078) and high Reynolds numbers and those obtained with small pipes (large d/D Ratios $\geq$ 0.17) and relatively low Reynolds numbers. In the first case $n \doteq \frac{1}{2}$, in the second case it is of the order of 1/4. This difference was confirmed by the results obtained from tests conducted on completely different installations and with different thermometric and mechanical measuring equipment. First the proper values of the index 'n' in equation (10.12) were determined from the results of the tests on the University $4''/1\frac{1}{2}''$ rig and the N.E.L. $20''/8''$ rig shown in Table (10.1). These values of n were

then used in equations (10.12) and (10.13) to predict the pocket error in the other two experiments which were conducted on 4"/3" and 17"/10" rigs. The predicted results were in good agreement with those obtained from the actual experiments (Table 10.2). The agreement of these results eliminates to a certain extent the possibility of a major experimental error. Thus it seems evident that a certain vital abnormality of the flow occurs between the two ranges of R_n and/or the two ranges of the ratios d/D.

The possibility that this anomaly is due to the shifting of the point of separation is ruled out because the pressure distribution curves (Fig.10.6) do not show any such major shift.

A possible reasoning is that at the large (d/D) ratios (≥ 0.17), due to the extreme restriction, the flow around the pocket is no longer isentropic. Thus the theory of the recovery factor based on the existence of an independent boundary layer on the pocket is no longer valid. There may exist a critical ratio (d/D) and a critical R_n between the two ranges. Unfortunately it was not possible to explore the intermediate zone between the two ranges with the available apparatus.

In addition to the research on conductive and non-conductive pockets suggested above, further research may be directed at developing pockets with minimum value of the parameter $(1 - P_c)$ in equation (10.12). The value of this parameter for a stream-line pockets is smaller than that for a cylinder, thus the stream-line pocket is likely to have a smaller overall recovery factor.

The maximum error in the results shown in Tables (10.1) and (10.2) is likely to be that of the temperature measurements. By considering the magnitude of the measured temperature difference and assuming a possible error of measurement of $\pm$ 1 mC^{o}, the average error in R_f is of the order of 5%. It is understandable however that the recovery correction is only a small fraction of the temperature difference and this error in its estimate is quite permissible.

10.10 Conclusions

The thermometer-pocket error may be determined directly for any pocket shape by the experimental method described in Section (10.2). If this is not possible a reasonable estimate of the error for cylindrical pockets may be obtained by the equations:

$$T_m - T_\infty = R_f \, (V^2/2g_u C_p) \qquad (10.13)$$

where

$$R_f = (1-P_c) \, P_r^n + P_c S_c \qquad (10.12)$$

P_r and S_c are properties of the fluid. P_c is typical of the pocket configuration; and n is dependent on the Reynolds number of the flow, on the shape of the pocket and on its aspect ratio i.e. the ratio of its diameter or width to the diameter of the pipe.

For circular cylindrical pockets P_c and n may be obtained from Table 10.1.

The investigation described in this Chapter is of a limited nature, and it cannot be over-emphasized that further full investigations are needed.

APPENDIX 10.1

The coefficient of Velocity distribution

$f = .025$	$R_n = .26 \times 10^5$	$K_v = 1.18$
$f = .020$	$R_n = .62 \times 10^5$	$= 1.14$
$f = .017$	$R_n = 1.36 \times 10^5$	$= 1.12$
$f = .015$	$R_n = 2.6 \times 10^5$	$= 1.11$
$f = .0125$	$R_n = 7.9 \times 10^5$	$= 1.105$
$f = .01$	$R_n = 2.5 \times 10^6$	$= 1.09$

$$\frac{U_\infty}{V_\infty} = 1 + 3.75\sqrt{f/8} + 2.5\sqrt{f/8}\log_e \frac{y}{r_o} \qquad (A.1.1)$$

(Streeter after Rouse, Ref. 22)

$$K_v = \frac{1}{r_o}\int_o^{r_o} (U_\infty/V_\infty)^2 dy = a^2 + 2b^2 - 2a\,b \qquad (A.1.2)$$

where $a = 1 + 3.75\sqrt{f/8}$

and $b = 2.5\sqrt{f/8}$

For $R_n < 8 \times 10^4$, $R^{\frac{1}{4}} = 0.317/f$ (Blasius Ref. 18)

For $R_n > 8 \times 10^4$, $2\log_{10} R_n = \frac{1}{\sqrt{f}} - \log f + 0.8$

(Von Karman univeral Law of resistance for smooth pipes Ref. 18)

$$f = (\tau_o/\tfrac{1}{8}\rho V^2)$$

From equation (A.1.1), $U_\infty = V_\infty$ at $y \doteq 0.25\ r_o$

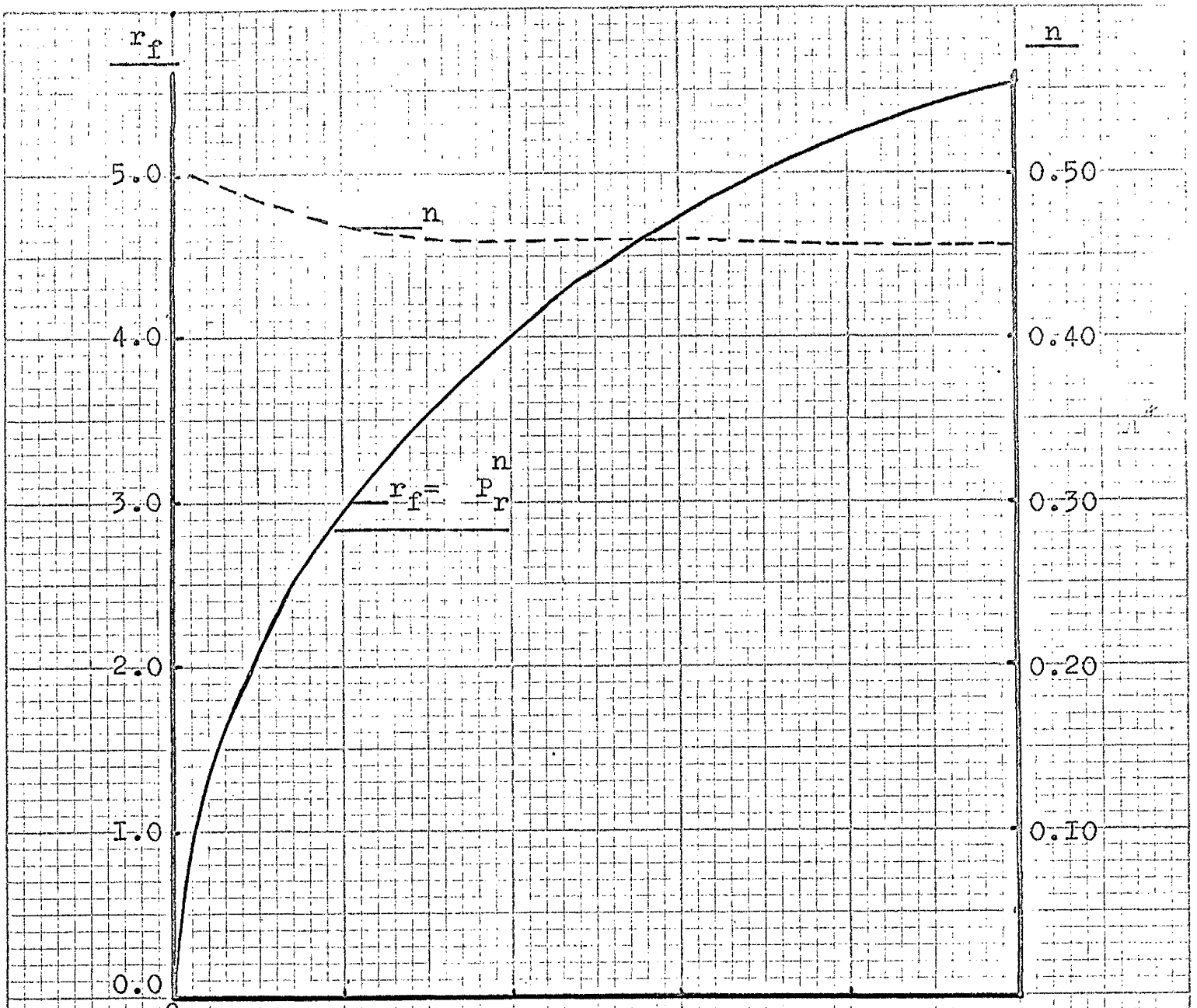


FIG.NO.10.1

VALUES OF r_f, THE FRICTIONAL RECOVERY FACTOR FOR LAMINAR BOUNDARY LAYER ON A FLAT PLATE.

(adapted from Eckert and Drewitz ref.)

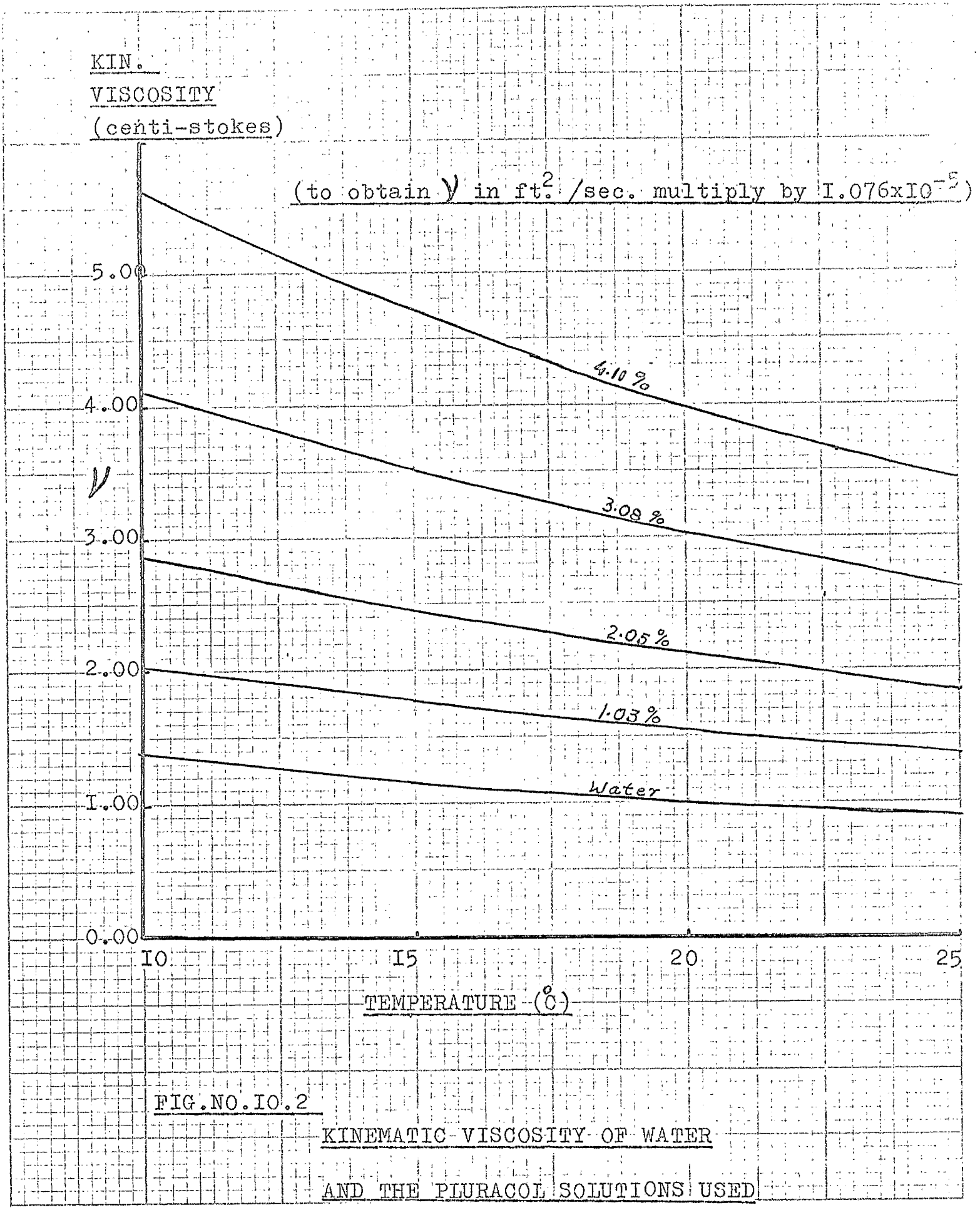


FIG.NO.10.2

KINEMATIC VISCOSITY OF WATER

AND THE PLURACOL SOLUTIONS USED

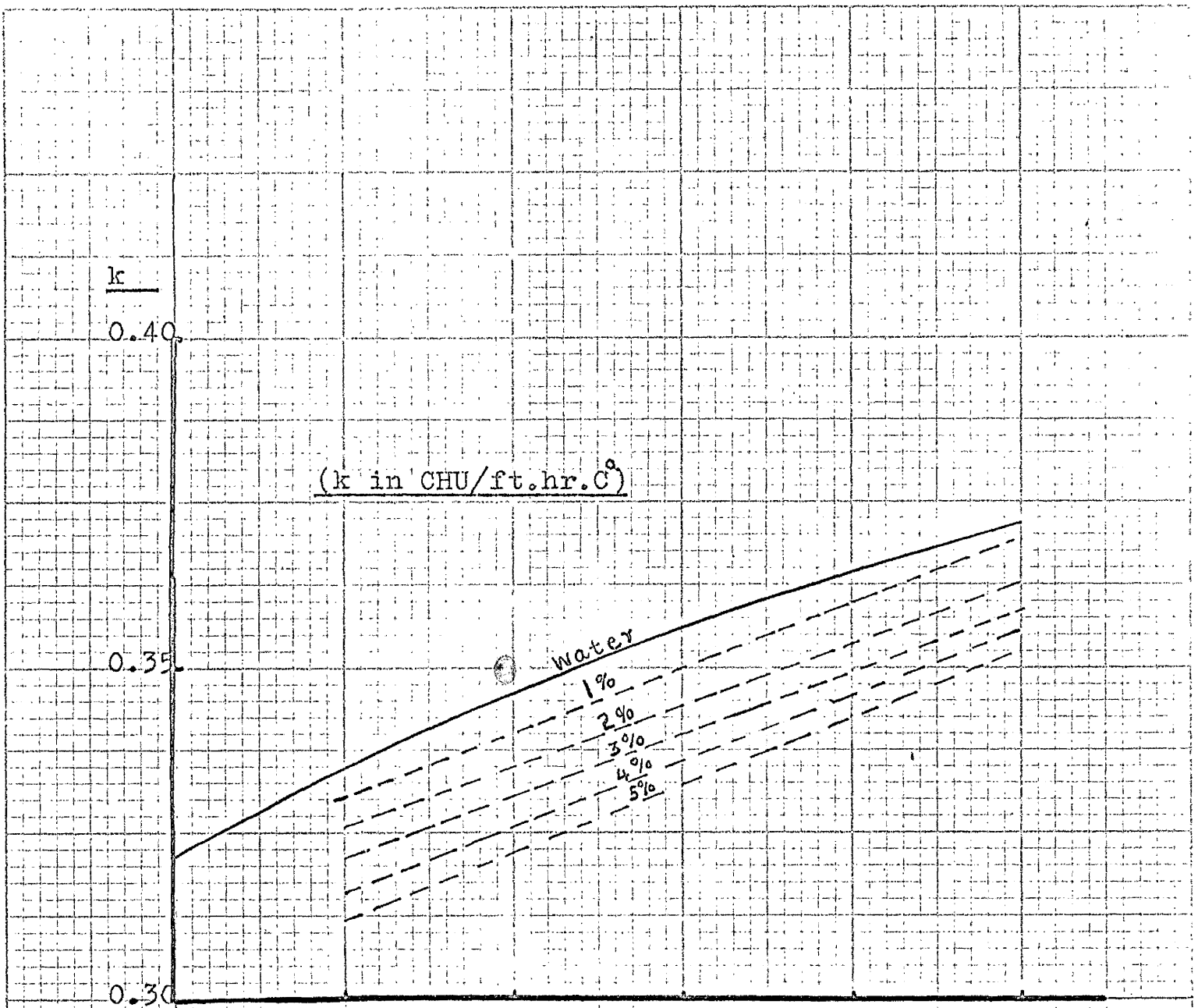


FIG.NO.10.3

THERMAL CONDUCTIVITY OF WATER

AND THE PLURACOL SOLUTIONS USED.

(adapted from measurements made by Vennart)

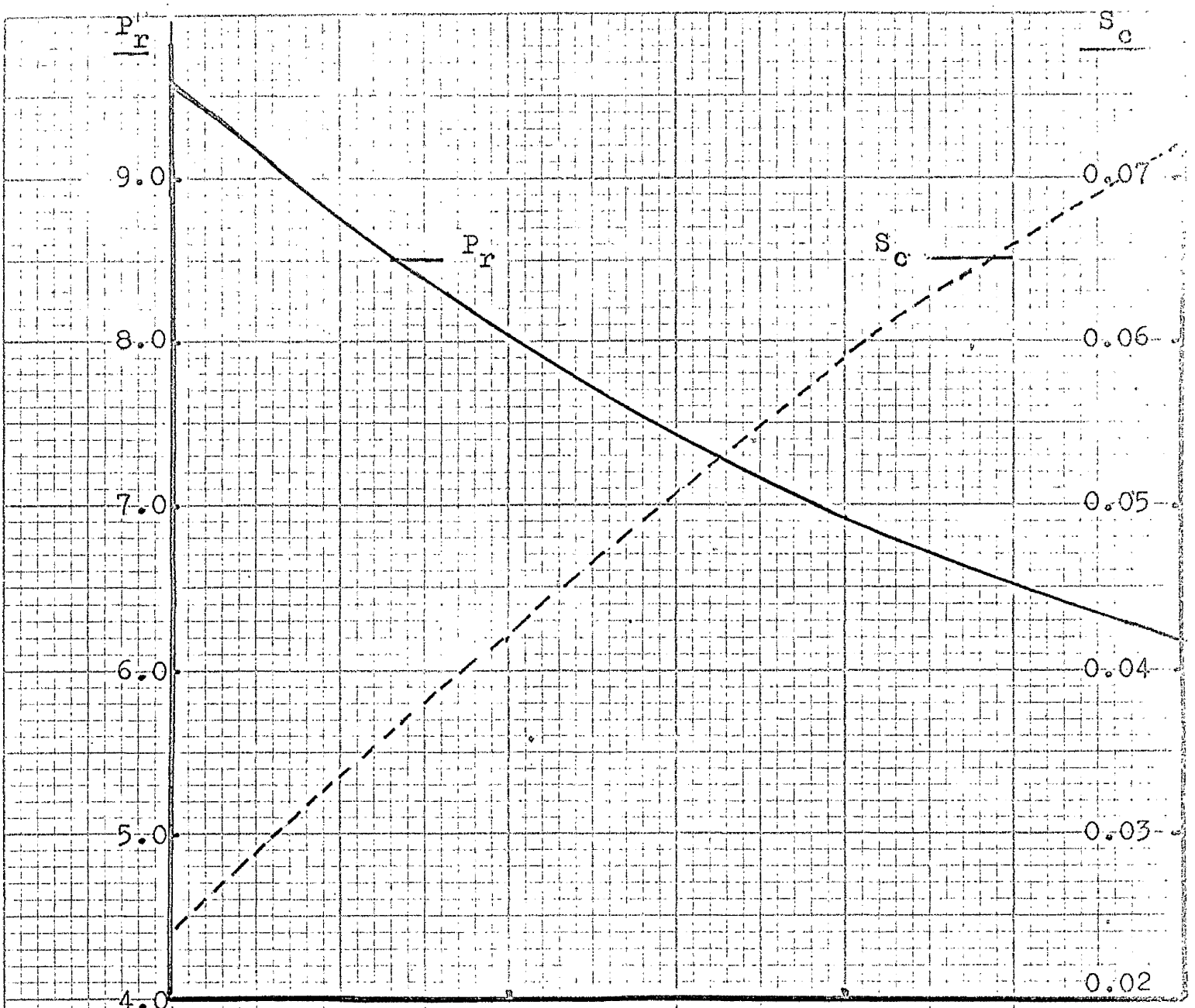


FIG.NO.10.4

THE STAGNATION COEFFICIENT

AND THE PRANDTL NUMBER OF WATER

(at atmospheric pressure)

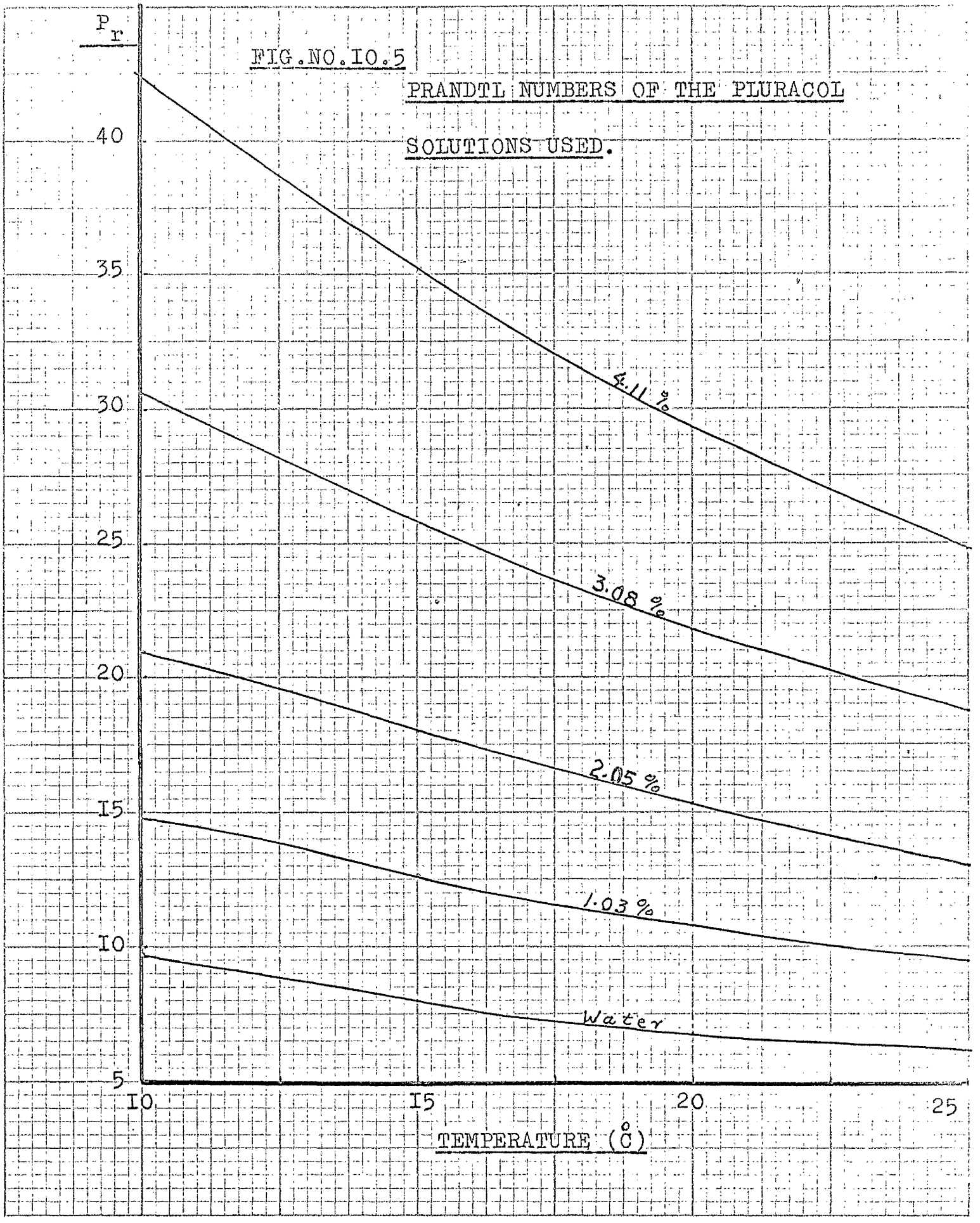


FIG.NO.10.5

PRANDTL NUMBERS OF THE PLURACOL SOLUTIONS USED.

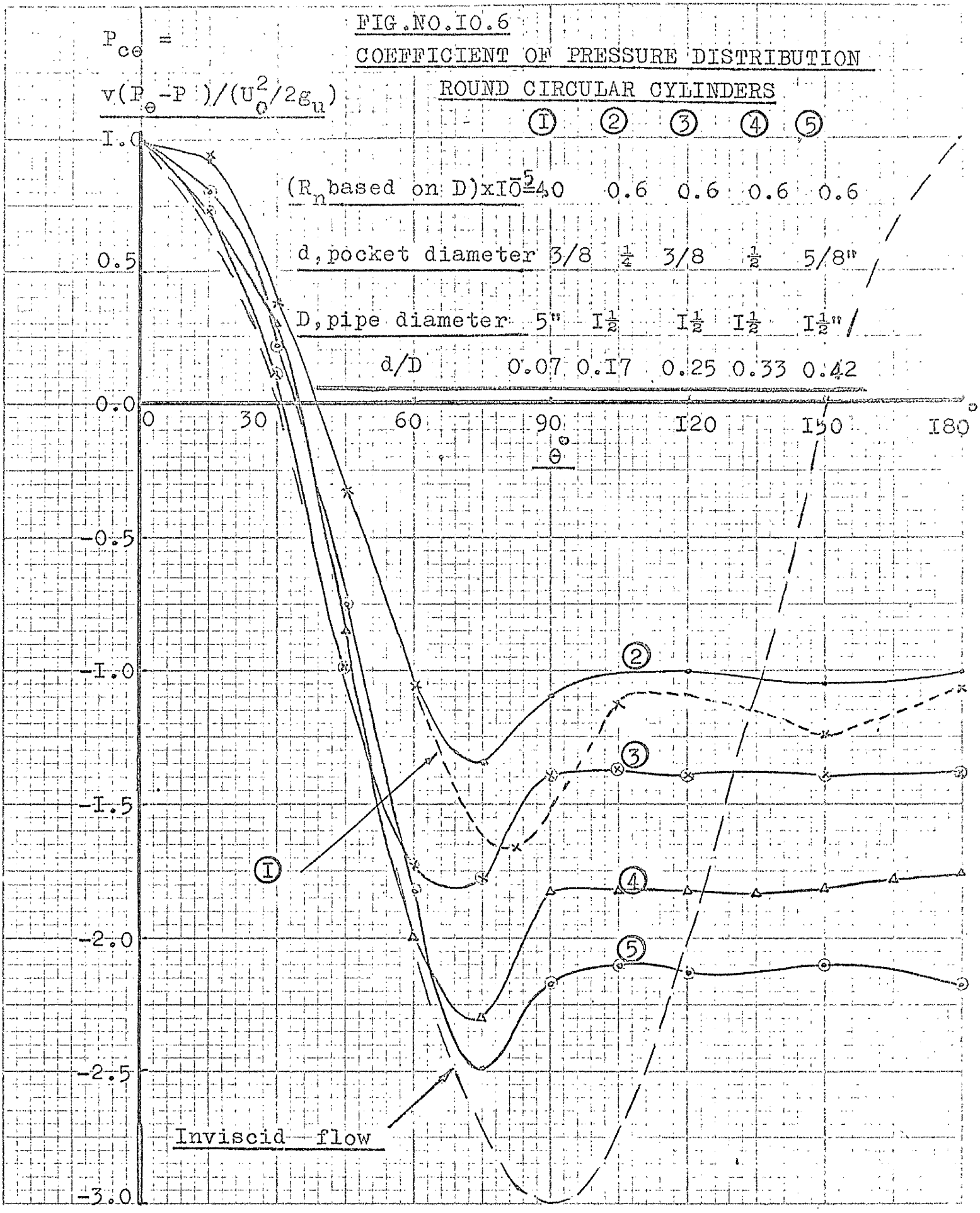


FIG.NO.10.6
COEFFICIENT OF PRESSURE DISTRIBUTION
ROUND CIRCULAR CYLINDERS
$P_{c\theta}$ =
$v(P_\theta - P)/(U_0^2/2g_u)$
(R_n based on D)x10^-5 40 0.6 0.6 0.6 0.6
d, pocket diameter 3/8 1/4 3/8 1/2 5/8"
D, pipe diameter 5" 1 1/2 1 1/2 1 1/2 1 1/2"
d/D 0.07 0.17 0.25 0.33 0.42
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
0 30 60 90 120 150 180
θ°
Inviscid flow

TABLE 10.1

THE OVERALL RECOVERY FACTOR FOR CIRCULAR CYLINDRICAL POCKETS

SUMMARY OF EXPERIMENTAL RESULTS

$$R_f = (1 - P_c) r_f + P_c S_c \qquad r_f = (R_f - P_c S_c) / (1 - P_c) = P_r^{\,n}$$

d/D_2			0.078			0.17			0.25			0.33			0.42		
Pocket diameter, d.			5"/8			1"/4			3"/8			1"/2			5"/8		
Average pressure coefficient, * P			- 0.75			- 0.63			- 1.05			- 1.26			- 1.46		
Solution	Av. P_r	Av. $R_n \times 10^{-5}$	R_f	r_f	n	R_f	r_f	n	R_f	r_f	n	R_f	r_f	n	R_f	r_f	n
N.E.L:- Water $V_2 \doteq$ 18 ft/sec D_1 = 20" D_2 = 8"	6.7	9.4 to 13.6	4.75	2.70	0.53												
UNIVERSITY:- Water $V_2 \doteq$ 22 ft/sec D_1 = 4" D_2 = 1½"	7.6	2.35				2.46	1.54	0.21	2.86	1.44	0.18	3.30	1.50	0.20	3.97	1.65	0.25
≒ 1% Pluracol	11.8	1.53				3.17	1.97	0.28	3.57	1.78	0.23	4.26	1.92	0.27	3.95	1.64	0.20
≒ 2% Pluracol	16.3	1.12				3.25	2.02	0.25	3.96	1.97	0.24	3.93	1.78	0.21	4.27	1.72	0.20
≒ 3% Pluracol	21.6	0.81				3.54	2.20	0.26	3.12	1.56	0.15	4.93	2.23	0.26	4.95	2.05	0.23
≒ 4% Pluracol	27.0	0.66				3.45	2.15	0.23	2.86	1.44	0.11	3.50	1.59	0.14	3.21	1.34	0.09

R_n is based on the pipediameter, D_2

R_f is based on the average velocity.

TABLE 10.2

VERIFICATION OF THE EQUATIONS (10.9.a) and (10.12)

$T_m = T + R_f V^2/2g_u C_p$ and $R_f = (1-P_c) P_r^n + P_c S_c$.

Test Data	T C°	Pr	R_n based on D_2 x 10^{-5}	H_v ft. of water	$\delta(\Delta T)_m$ (measured) m C°	$\delta(\Delta T)$ calculated by above equations m° C	
Test on 4"/3" Pump	12.2	8.9	2.3	2.2	4.5	4.0	
D_1 = 4" D_2 = 3" d = ½"	12.25	8.8	2.8	3.2	6.4	5.9	From N.E.L. Report No. 130
d/D = 0.17	12.35	8.8	3.2	4.2	8.1	7.7	
P_c = - 0.63) Table	12.45	8.7	3.5	5.1	9.6	9.3	
n = 0.21) 10.1	12.55	8.7	4.0	6.4	11.5	11.6	
Test on the reversible Pump/turbine model	Average of five readings at the same rate of flow						
D_1 = 17.35" D_2 = 10.4"							
d = 5"/8 d/D_2 = 0.06							
P_c = - 0.75) Table	20.9	6.8	7.5	1.54	5.0	5.2	Average R_f = 4.75
n = 0.53) 10.1							

PART C

THE USE OF TEMPERATURE AS A VARIABLE IN PRACTICAL HYDRAULICS

CHAPTER ELEVEN

THE THERMODYNAMIC METHODS OF MEASURING THE EFFICIENCY OF HYDRAULIC MACHINES

11.1 Classification and theory of the methods

These methods may be classified as following:-

1) The Thermometric Methods:-

(a) The Direct Thermometric Method

and (b) The Partial Thermometric Method

11) The Null Method

1) The Thermometric Methods:-

(a) The Direct Thermometric Method -

In general, all the Thermometric Methods involve quantitative measurement of temperature difference. The Direct Thermometric Method involves 'direct' measurement of The temperature difference across the machines. This was the method originally suggested by Poirson (Ref. 23) and adopted by all the early experimenters. For this method equations (4.11) and (4.13) may be directly applied.

(b) The Partial Thermometric Method:-

In this method a sample of water from the high-pressure side (e.g. of a turbine) is throttled to P_3, where $P_3 = P_2$, the pressure at the downstream side, and the temperature difference $(T_3 - T_2)$ is measured (Fig. 11.1).

From the information available this method was used for turbines only, but a similar method may be adopted for pumps.

The Theory of the Partial Thermometric method is as follows. Assuming that the path of the water from sections (1) to section (3) in Fig. (11.1.a) is perfectly insulated against heat transfer to or from the surroundings, and since there is no external work done on (or by) the water in its passage between the two sections, then by the law of conservation of energy, the total energy per unit mass at section (1) is the same as that at section (3). Hence for turbines (Fig. 11.1.a)

$$(e_1 - e_2) = (e_3 - e_2) \qquad (11.0)$$

Thus in the numerator of equation (4.11), the suffix '3' may replace the suffix '1' and equation (4.11) may be written as:-

$$\eta_{ht} = \frac{\frac{g_u}{g} A_{T_3}(P_3-P_2) + \frac{g_u}{g} C_{p_2}(T_3-T_2) + \left(\frac{V_3^2-V_2^2}{2g}\right) + (Z_3-Z_2)}{\frac{g_u}{g} A_s(P_3-P_2) + \left(\frac{V_1^2 - V_2^2}{2g}\right) + (Z_1 - Z_2)} \qquad (11.1)$$

If $P_3 = P_2$, equation (11.1) becomes

$$\eta_{ht} = \frac{\frac{g_u}{g} C_{P2}(T_3-T_2) + \frac{(V_3^2-V_2^2)}{2g}\frac{}{2g} + (Z_3 - Z_2)}{\frac{g_u}{g} A_s (P_1-P_2) + \frac{(V_1^2-V_2^2)}{2g}\frac{}{2g} + (Z_1 - Z_2)} \qquad (11.2)$$

Here the need for A_T is obviated.

Also for pumps, (Fig. 11.1.b)

$$(e_2 - e_1) = (e_3 - e_1) \qquad (11.3)$$

Thus in the donominator, of equation (4.13), the suffix '3' may replace the suffix '2' and equation (4.13) may be written as

$$\eta_{hp} = \frac{\frac{g_u}{g} A_s(P_2-P_1) + \left(\frac{(V_2^2 - V_1^2)}{2g}\frac{}{2g}\right) + (Z_2 - Z_1)}{\frac{g_u}{g} A_{T_1}(P_3-P_1) + \frac{g_u}{g} C_{p_3}(T_3-T_1) + \frac{(V_3^2-V_1^2)}{2g}\frac{}{2g} + (Z_3-Z_1)} \qquad (11.4)$$

For pumps P_3 = atmospheric pressure, while P_1, is usually less than atmospheric. Thus P_3 and P_1 cannot be made equal. The value of A_T, however, may be taken as that corresponding to atmospheric pressure,

since the small difference between P_3 and P_1 has a negligible effect on A_T.

In the partial thermometric method, the temperature difference, (T_3-T_2) for turbines and (T_3-T_1) for pumps, is large compared with (T_2-T_1) which is obtained if the direct thermometric method is used. However, the effect of any inaccuracy in the measurement of temperature is ultimately the same, whichever of the two methods is used.

The partial thermometric method is useful when it is needed to avoid insertion of the thermometer in the high pressure side of the machine where it is susceptible to damage by vibrations; it can then be conveniently placed at P_3.

(ii) <u>The Null Method</u>

This method is extensively developed and used on turbines by the engineers of E.d.F. in France (Ref.5) and by Thom (of The University of Glasgow) and collaborators (Refs. 11,24 & 25). Like the partial thermometric method, it involves throttling a sample of water drawn from upstream of the machine; but here, the water is throttled to the downstream temperature instead of to the downstream pressure, and so

$$T_3 - T_2 = 0 \quad \text{(Fig. 11.1.a)}$$

As before the suffix '3' may replace the suffix '1' in the numerator and equation (4.11) may be written as

$$\eta_{ht} = \frac{\frac{g_u}{g} A_T(P_3-P_2) + \frac{(V_3^2-V_2^2)}{2g} + (Z_3 - Z_2)}{\frac{g_u}{g} A_S(P_1-P_2) + \frac{(V_1^2-V_2^2)}{2g} + (Z_1 - Z_2)} \qquad (11.5)$$

In this equation the need for C_p is obviated.

There is no equivalently simple method for Pumps.

11.2 Temperature Measurement

As a guide to the order of temperature differences obtainable across hydraulic machines, the following equations may be used for water at normal temperatures.

For a turbine,

$$\Delta T \doteq \frac{H}{J} \times \frac{g}{g_u} (1-\eta_t) \quad C^\circ \qquad (11.8)$$

For a pump,

$$\Delta T \doteq (\frac{1}{\eta_p} - 1) \frac{H}{J} \times \frac{g}{g_u} \quad C^\circ \qquad (11.9)$$

By differentiating the above equations the error in the efficiency measurement, $\delta(\eta)$, resulting from an error, $\delta(\Delta T)$, in the measurement of ΔT, may be approximately expressed by:

For a turbine, $$\delta(\eta_t) = -\frac{g_u}{g} \times \frac{J}{H} \times \delta(\Delta T) \qquad (11.10)$$

For a pump, $$\delta(\eta_p) = -\frac{g_u}{g} \times \frac{J}{H} \times \eta_p^2 \delta(\Delta T) \qquad (11.10)$$

Thus in a turbine of 140 ft head, having an efficiency of 90%, from equation (11.7), the temperature rise is equivalent to about 0.01 C^o; and for an accuracy of 1% in the measurement of the efficiency of this machine, the required accuracy in the measurement of $(T_2 - T_1)$ is

$$1\% \times \frac{140}{1400} = 0.001\ C^o,$$ assuming that g = 32.174 ft/sec^2.

These figures indicate the extreme care needed with such measurements. They also show that the higher the head of the machine the less is the required accuracy in temperature measurements.

11.3 Comparison of the various thermodynamic methods of measuring the hydraulic efficiency

Since 1954 when Willm and Campmas proposed it (Ref. 5), the Null method became by far the most popular of the forementioned methods. Its main attraction lies in the fact that no quantitative

measurement of the temperature is necessary (as $T_3-T_2 = 0$). Thus although the temperature measuring apparatus needs to be equally stable and sensitive, absolute calibration of the thermometers is not necessary. The thermometers need only have matched characteristics.

The main drawback common to both the null and the partial thermometric method, is that since the amount of sampled water is small, the effect of heat transfer from the surroundings is considerable, and unless the sampling conduits and the calorimeters are carefully lagged, the results may be quite erratic. On the other hand, in the direct thermometric method, the difference in the velocity head $(V_1^2-V_2^2)/2g$ may be very large, necessating the use of a pocket correction as explained in Chapter 10.

However on the surmounting of this problem, and in the development of such cheap and accurate bridges as these described in Chapters (7) and (9), the direct thermometric method may be much preferred to the other two methods. In fact all the efficiency tests carried out in this research are done by this method with most satisfactory results. Only one test is done by the partial thermometric method for

verification. The main reasons for the preference of the direct thermometric method may be outlined as following:-

(1) The direct measurement of temperature obviates the main source of error and uncertainty which is incurred in case of the other methods, namely heat transfer to the branch pipe and to the calorimeter.

(2) In case of pumps the null method cannot be applied. Brand did describe a method of applying it (unpublished) but this involved very complicate auxiliary hydraulic and electric circuits.

(3) All the machines tested are relatively small. Thus it is advantageous to use the direct thermometric method; because it will be possible to make the thermometer pockets span the whole pipe and hence integrate the fluid temperature.

(4) In certain cases e.g. in the turbine test No. 5, 8 and 10 listed in Table (11.1), where the water temperature is relatively high ($22^{\circ}C$), the water temperature at

the outlet is nearly equal to that at the inlet, because the cooling of water due to expansion is nearly equivalent to the heating due to friction. Thus the null method cannot be applied because the sampled water, by the time it reaches the calorimeter, will have a temperature T_3, higher than the outlet temperature T_2.

(5) The main difficulty which otherwise may have handicapped the use of the direct thermometric method (namely the difficulty of accurate measurement of the small temperature differences involved in hydraulic flow) has already been successfully overcome, or at least considerable progress has been achieved in this direction, by the development of the techniques described in Chapters (7) and (9).

11.4 Efficiency tests

Ten efficiency tests were carried out on three machines of different sizes and capacities.

Details of these tests are shown on Table (11.1). The machines tested are:-

i) 4"/3" 160 ft. Centrifugal pump

This is a single stage machine in the Fluid Mechanics Laboratory of the University of Glasgow. It has a double entry impeller, $11\frac{3}{4}$ inches and diameter and an outlet width of 15"/32 inch and is rated at 160 ft head, 202 gallon per minute at 1750 r.p.m.

The pump is driven by a variable speed a.c. motor, the output torque of which is measured by a swinging-field dynamometer.

All measuring apparatus e.g. tachometer and dynamometer balance were carefully checked and/or calibrated beforehand.

ii) $2\frac{1}{2}$"/$2\frac{1}{2}$", 1155 ft., multi-stage centrifugal pump

This is a 12-stage pump driven by a constant speed motor (2950 r.p.m.). The machine is installed in the Fluid Mechanics Division of the National Engineering Laboratory (N.E.L.), East Kilbride, Glasgow. Since the pump was not fitted with a dynamometer the input to the pump had to be worked out from the input wattage and the efficiency of the motor. These were obtained from the manufacturer's curves.

iii) The pump/turbine model (Fig. 11.9)

This is a (1/6.88) model of a reversible pump/turbine industrial machine. The model is installed in the Hydraulic Research Laboratory of Messrs. English Electric Co. Ltd., Rugby, who kindly invited the University of Glasgow research team to carry out tests on the model. All conventional measurements of the pressure and the rate of flow, were undertaken by the English Electric staff - using English Electric standard equipment.

The thermometric tests were conducted by both the a.c. and the hydraulic bridge, and for both pump and turbine settings of the machine.

In case of the 4"/3" pump and the pump/turbine model, it was necessary to make thermometer pocket correction, because the sizes of the inlet and outlet pipes are different (10"/17" for the model). The correction was determined experimentally by the method described in Chapter 10. The same value might have been correctly predicted by the semi-empirical equation No.(10.9.a).

Test No. (2) on the 4"/3" pump was carried out with the machine totally insulated with polyzote granules. This provided no substantial improvement on the other

tests carried out without thermal insulation. Thus it may be safely concluded that heat transfer is hardly effective, or that the error due to it is much less than the other inherent experimental errors.

Concurrently with all these thermometric tests, conventional tests were conducted for comparison. The percentage differences between the two results are shown on Table (11.1). It is worth noting that the same values of head were used in the equations of both methods. Thus these differences were actually due to the differences in accuracy of the measurement of temperature on one hand, and of the rate of flow and the external work on the other.

DISCUSSIONS AND CONCLUSIONS

The thermodynamic methods provide a cheap and simple means for measurement of the efficiency of hydraulic machines; they do not call for any undue interruption of the working of the machines and are most useful for the current checking of their performance; it is necessary only to make provision for the thermometer pockets on erection of the plant and this provides a permanent means of checking the performance.

The thermodynamic methods are most useful for performance tests, because, unlike the acceptance tests which require measurements of the absolute values of the efficiency, performance tests require detection only of any change in value of efficiency relative to previous observations. This requirement can be successfully satisfied by consistent testing using the thermodynamic methods, or even by only determining the hydraulic losses by equation 4.15.

An advantage of these methods is that they make unnecessary the costly 'detour' involving accurate measurement of the discharge and the electrical power exchange and the estimation of the efficiency of generators or motors, necessary when testing by the conventional methods.

The development of the techniques of measuring small temperature differences, described in Part B (i.e. the a.c. bridge, the pair ratio and the hydraulic bridge) may make the thermodynamic methods comparable in accuracy with any of the other established methods, for heads considerably lower than hitherto considered possible. For example most of the international test codes stipulate that the thermodynamic methods are acceptable only at heads of the order of 328 ft. or more. Some of the tests described in this chapter were conducted at heads of about 100 ft. The results obtained were in very good agreement with those obtained by conventional methods. Admittedly these thermodynamic tests were conducted at laboratory conditions, but so were the concurrent conventional tests with which they were compared. These are themselves susceptible to large field errors.

TABLE NO. (11.1)

LIST OF EFFICIENCY TESTS conducted by the RESEARCH TEAM (*)

Test No.	Machine	Date of Test	Place of Test	Thermometric		Conventional		Table of Results (T) and efficiency curves (C)	Average error ± %
				Temperature measurement	Pressure measurement	Rate of flow	External work, w		
1	4/3" 160 ft. centrifugal Pump. N.T.I.* Test head ≐ 100 ft.	April 1963	University of Glasgow	A.C. bridge and the pair ratio method B.T.T.*	Mercury manometer	By weighing Tank	Swinging-field dynamomter	(T) No. 11.2 (C) Fig. No. 11.2	< 1.0
2	The same machine. The machine is totally insulated with polyzote granules.	May 1963	"	"	"	"	"	(T) No. 11.3 (C) Fig. No. 11.3	< 0.5
3 and 4	Same machine N.T.I. Test by the hydraulic bridge	May 1964	"	Hydraulic Bridge P.I.P.	"	"	"	(T) No. 11.4 and 11.5 (C) Fig. No. 11.4	< 1.0 < 1.5
5 and 6	$2\frac{1}{2}$"/$2\frac{1}{2}$" .1155 ft. 12 - stage centrifugal pump - N.T.I. Direct and partial thermometric methods	August 1963	N.E.L.*	A.C. bridge and the pair ratio method B.T.T.	Calibrated Bourden Gauges	"	Estimated from motor input and efficiency	(T) No. 11.6 and 11.7 (C) Fig. No. 11.5 and 11.6	< 1.0
7 and 8	Model of reversible pump/turbine pump and turbine tests. 130-100 ft. N.T.I.	May 1967 4	English Electric Co. Ltd., Rugby.	" P.I.P.	Mercury manometer	Venturi-meter	Swinging-field dynamometer	(T) No. 11.8 (C) Fig. No. 11.7	< 1.0
9 and 10	" Pump and turbine tests by the hydraulics bridge	May 1967 4	"	Hydraulic bridge P.I.P.	"	"	"	(T) No. 11.9 (C) Fig. No. 11.8	< 2.5 Ψ < 2.0

* N.T.I., No thermal insulation

N.E.L., The National Engineering Laboratory, East Kilbride, Glasgow.

B.T.T. Barber type thermometers.

Ψ In this test the detector behaved erratically

(*) Refer to acknowledgements in the Preface.

P.I.P. Platinum in Pyrex thermometers (Chapter 6).

TABLE NO. 11.2

RESULTS OF TEST NO. 1 ON 160 FT., 4 INCH/3 INCH CENTRIFUGAL PUMP *

(DIRECT THERMOMETRIC METHOD) TEST BY THE a.c. BRIDGE.

Test No	Mechanical measurements: N = 1520 rev/min (constant); $H = A_s (P_2 - P_1) \times (g/g_u)$							Electrical measurements: Current through each thermometer = 1 mA; Frequency = 400 c/s				Efficiency	
				Thermometric		Conventional							
	Water rate of flow Q (gal/min)	Head H (ft)	$H_v = \frac{V_2^2 - V_1^2}{2g}$ (ft)	Input per unit weight of water $w_u \times (g_u/g)$ (ft)	Ideal input per unit weight of water $w_i \times (g_u/g)$ (ft)	Input (hp)	Ideal Input (hp)	Inlet water temperature (°C)	Measured Δt (mdegC)	Pocket correction $-\delta(\Delta t)$ (mdegC)	Corrected Δt (mdegC)	Pump efficiency by the thermometric method (Per cent)	Pump efficiency by the conventional method (Per cent)
1	92	108.0	0.3	271.4	108.3	7.15	3.02	12.00	119.9	- 1.0	118.9	39.9	42.2
2	139	106.0	0.6	209.7	106.6	8.89	4.49	12.05	77.8	- 1.8	76.0	50.8	50.5
3	183	106.5	1.0	189.0	107.5	10.53	5.96	12.10	63.1	- 2.5	60.6	56.9	56.5
4	229	105.5	1.6	176.8	107.1	12.10	7.43	12.15	55.8	- 3.6	52.2	60.6	61.4
5	265	102.0	2.2	165.3	104.2	13.14	8.37	12.20	50.5	- 4.5	46.0	63.0	63.7
6	321	98.0	3.2	154.8	101.2	15.05	9.84	12.25	46.9	- 6.4	40.5	65.4	65.4
7	367	94.5	4.2	149.8	98.7	16.58	10.98	12.35	46.3	- 8.1	38.2	65.9	66.2
8	407	88.0	5.1	143.2	93.1	17.83	11.48	12.45	47.4	- 9.6	37.8	65.0	64.4
9	454	78.0	6.4	141.1	84.4	19.39	11.61	12.55	53.8	-11.5	42.3	59.8	59.9

Thermometer X: R_o = 22.7826 ; a_o = 0.003 927 08/°C; δ = 1.4918 . Thermometer Y: R_o = 25.3526 ; a_o = 0.003 925 9/°C; δ = 1.4910

* N.E.L. Report No. 130 by Foord, Langlands and Elagib.

TABLE NO. 11.3.

RESULTS OF TEST Nr. 2 on 160 ft. 4"/3" CENTRIFUGAL PUMP INSULATED WITH "POLYZOTE." *

(DIRECT THERMOMETRIC METHOD). TEST by the a.c. BRIDGE

TEST NO.	CONVENTIONAL MEASUREMENTS					THERMOMETRIC MEASUREMENTS				EFFICIENCY	
	Q GAL/MIN.	HEAD $H = A_s (P_2 - P_1) \times (g_u/g)$ ft.	H_v	PUMP INPUT (HP.)	IDEAL INPUT (HP.)	T_1 °C	CORRECTED Ψ ΔT m C°	INPUT W $\times (g_u/g)$ ft.	IDEAL INPUT W $\times (g_u/g)$ ft.	η_p (thermometric) %	η_p (conventional) %
1	440	85.4	6.1	19.33	12.20	15.6	40.9	144.8	91.5	63.2	63.1
2	403	91.1	5.0	17.94	11.73	15.8	39.3	147.2	96.1	65.3	65.4
3	324	100.4	3.3	15.66	10.18	15.9	42.4	158.3	103.7	65.5	65.0
4	263	106.5	2.1	13.58	8.65	15.9	47.7	170.2	108.6	63.8	63.7
5	188	109.1	1.2	10.85	6.28	15.9	62.3	192.5	110.3	57.3	57.9

Constant speed : 1520 r.p.m.

T_R = 18 - 20°C

* Polyzote : refer to Chapter 8.

Ψ Correction according to Table 11.2.

TABLE NO. 11.4

RESULTS OF TEST NO. 3 ON 160 FT. 4"/3" CENTRIFUGAL PUMP

(DIRECT THERMOMETRIC METHOD) TEST BY THE HYDRAULIC BRIDGE (1)

TEST NO.	CONVENTIONAL MEASUREMENTS						THEROMETRIC MEASUREMENTS				EFFICIENCY	
	SPEED N r.p.m.	Q GAL/MIN	HEAD $H = A_s(P_2-P_1) \times (g_u/g)$ ft.	H_v ft.	PUMP INPUT HP	IDEAL PUMP INPUT HP	T_1 °C	CORRECTED ΔT mC°	INPUT W $\times (g_u/g)$ ft.	IDEAL INPUT $W_i \times (g_u/g)$ ft.	(thermometric) %	(Conventional) %
1	1328	150	81.0	0.7	6.90	3.71	14.8	56.4	152.7	81.7	53.5	53.8
2	1510	167	105.0	0.9	10.10	5.35	14.9	70.7	196.1	105.9	54.0	53.0
3	1500	216	102.2	1.5	11.50	6.76	14.9	56.5	175.1	103.7	59.2	58.8
4	1490	232	100.9	1.7	11.91	7.21	15.0	52.4	168.2	102.6	61.0	60.5
5	1478	252	98.1	2.0	12.30	7.64	15.1	49.4	162.2	100.1	61.7	62.1
6	1510	273	103.0	2.3	13.70	9.38	15.2	49.4	166.6	105.3	63.2	64.0

The bridge constant C = 20.7 m C° (average) T_R = 18 - 19° C

Ψ Correction according to Table 11.2.

TABLE NO. 11.5

RESULTS OF TEST NO. 4 on 160 FT. 4"/3" CENTRIFUGAL PUMP (ii)

(DIRECT THERMOMETRIC METHOD) TEST BY THE HYDRAULIC BRIDGE

TEST NO.	CONVENTIONAL MEASUREMENTS					THERMOMETRIC MEASUREMENTS				EFFICIENCY	
	Q GAL/MIN.	HEAD $H = \frac{A_s(P_2-P_1)}{x\,(g_u/g)}$ ft.	H_v ft.	PUMP INPUT HP.	IDEAL PUMP INPUT HP.	T_1 °C.	CORRECTED ΔT m C° $^{\Psi}$	INPUT W $x(g_u/g)$ ft.	IDEAL INPUT W_i $x(g_u/g)$ ft.	(THERMOMETRIC) %	(CONVENTIONAL) %
1	222	104.2	1.7	12.15	7.12	15.6	53.0	177.1	105.9	59.8	58.6
2	269	100.8	2.3	13.57	8.40	15.9	44.7	163.4	103.1	63.1	61.9
3	302	99.1	2.8	14.65	9.32	15.9	41.2	158.0	101.9	64.5	63.6
4	343	96.0	3.6	15.94	10.35	16.0	36.0	149.1	99.6	66.8	64.9
5	373	92.3	4.3	16.64	10.92	16.0	34.4	144.6	96.6	66.8	65.6
6	415	88.0	5.3	17.93	11.73	16.1	32.4	139.9	93.3	66.7	65.4
7	449	82.5	6.2	18.62	12.07	16.4	32.3	136.0	88.7	65.2	64.8

The bridge constant C = 21.9 m C° (Average) $T_R \doteq 19°C.$

$^{\Psi}$ Correction according to Table 11.2 Constant speed = 1510 r.p.m.

TABLE NO. 11.6

TEST NO. 5 ON 12-STAGE, 1155 FT., $2\frac{1}{2}$ INCH / $2\frac{1}{2}$ INCH CENTRIFUGAL PUMP *

(DIRECT THERMOMETRIC METHOD) TEST BY THE a.c. BRIDGE

Test No.	Speed (rev/min)	Water rate of flow (gal/min)	Head (ft)	Thermometric: Pump input per unit weight of water $W \times (g/g)^u$ (ft)	Thermometric: Pump Ideal input per unit weight of water $W_i \times (g/g)^u$ (ft)	Conventional: Pump input (hp)	Conventional: Ideal Pump input (hp)	Inlet water temperature C^0	Measured Δt (C^0)	Efficiency: Pump efficiency by the thermometric method (per cent)	Efficiency: Pump efficiency by the conventional method (per cent)
1	2968	35.8	1438	3145	1436	34.11	15.57	22.5	1.292	45.7	45.6
2	2971	53.0	1453	2552	1453	40.34	23.32	22.4	0.856	56.9	57.8
3	2972	56.5	1432	2446	1432	41.84	24.50	22.3	0.794	58.5	58.6
4	2962	68.2	1407	2210	1407	44.93	29.06	22.6	0.642	63.7	64.7
5	2963	74.0	1388	2107	1388	47.15	31.10	22.2	0.581	65.9	66.0
6	2959	84.5	1350	1971	1350	50.19	34.54	22.2	0.509	68.5	68.8
7	2954	95.6	1310	1857	1310	53.04	37.92	22.2	0.454	70.5	71.5
8	2953	105.2	1266	1768	1266	55.80	40.33	22.3	0.420	71.6	72.3
9	2956	119.0	1182	1652	1182	59.37	42.59	22.4	0.393	71.5	71.7

Thermometer X: $R_o = 24.3659$; $a_o = 0.003\ 926\ 75/^0C$; $\delta = 1.4924$. Thermometer Y: $R_o = 23.8528$; $a_o = 0.003\ 926\ 61/^0C$; $\delta = 1.4921$.

* N.E.L. Report No. 130 by Foord, Langlands and Elagib.

TABLE No. 11.7

RESULTS OF TEST NO. 6 ON 12-STAGE, 1155 FT. $2\frac{1}{2}$"/$2\frac{1}{2}$" CENTRIFUGAL PUMP

(PARTIAL THERMOMETRIC METHOD) TEST BY THE a.c. BRIDGE

	MECHANICAL MEASUREMENTS						THERMOMETRIC MEASUREMENTS			EFFICIENCY	
TEST NO.	SPEED N r.p.m.	Q GAL/MIN.	H HEAD $A_s(P_2-P_2)$ x (g_u/g) ft.	PUMP INPUT HP.	IDEAL PUMP INPUT HP.	HEAD * $H = A_s(P_3-P_1)$ x (g_u/g) ft.	$\Delta T = T_3 - T_1$ C^o	INPUT W x (g_u/g) ft.	IDEAL INPUT W_i x(g_u/g) ft.	(THERMOMETRIC) %	(CONVENTIONAL) %
1	2948	119.5	1165	59.08	42.19	316	0.957	1634	1165	71.3	71.4
2	2956	104.8	1255	55.75	39.86	215	1.103	1744	1255	72.0	71.5
3	2963	94.5	1303	53.19	37.31	171	1.221	1868	1303	69.8	70.1
4	2966	85.1	1351	50.48	34.84	123	1.309	1946	1351	69.4	69.0
5	2964	74.5	1391	47.05	31.40	84	1.432	2082	1391	66.8	66.7
6	2967	63.1	1410	43.38	26.96	48	1.584	2261	1410	62.4	62.1
7	2971	48.8	1449	38.70	21.43	10	1.861	2613	1449	55.5	55.4

* Refer to Fig. 11.1.

T_R = 18.5 - 20.5 °C.

TABLE NO. 11.8.

RESULTS OF TESTS NO. 7 and 8 ON REVERSIBLE - PUMP/TURBINE MODEL

(DIRECT THERMOMETRIC METHOD) TEST BY THE a.c. BRIDGE

	PUMP TEST (CONSTANT SPEED TEST)					TURBINE TEST (CONSTANT RATE OF DISCHARGE)				
TEST NO.	T_1 °C	TOTAL HEAD $H + H_v$ ft.	Q/Q max.	CORRECTED ΔT $(T_2 - T_1)$ m C°	THERMOMETRIC η_p % - CONVENTIONAL η_p %	T_1 °C	N/N max	TOTAL HEAD $H + H_v$ ft.	CORRECTED ΔT $(T_2 - T_1)$ m C°	THERMOMETRIC η_p % - CONVENTIONAL η_p %
1	21.8	116.0	1.00	17.0	+0.8 +2.4	20.1	0.88	99.8	4.2	+0.3 -0.5
2	21.8	118.5	0.98	17.7	-0.1 +1.2	20.2	0.79	98.0	5.1	+0.7 -0.3
3	21.9	121.3	0.96	18.1	-0.5 +0.6	20.2	0.80	97.0	3.9	+1.3 -0.4
4	22.0	125.0	0.91	18.1	-0.2 +0.9	20.3	0.86	99.0	3.1	+0.7 +0.8
5	22.1	128.3	0.89	17.2	+0.8 +1.3	20.4	0.89	100.2	3.7	+0.9 +0.0
6	22.1	130.8	0.87	17.2	+0.9 +1.4	20.5	0.92	101.0	4.2	+0.9 +0.1
7	22.1	134.1	0.84	19.7	-1.0 -0.9	20.5	0.95	101.2	4.7	+0.9 +0.2
8	22.2	140.9	0.77	20.7	-0.5 -0.7	20.6	1.00	101.1	4.4	+1. +0.8
9	22.2	146.8	0.61	35.0	-0.5 +1.2					

T_R = 22 - 23 °C $Z_2 - Z_1$ = 3.28 ft. T_R = 19.5 - 20.5 °C

* correction according to equation (10.12) and Tables (10.1) and (10.2)

TABLE NO. 11.9

RESULTS OF TESTS NO. 9 and 10 ON REVERSIBLE - PUMP/TURBINE MODEL

(DIRECT THERMOMETRIC METHOD) TEST BY THE HYDRAULIC BRIDGE.

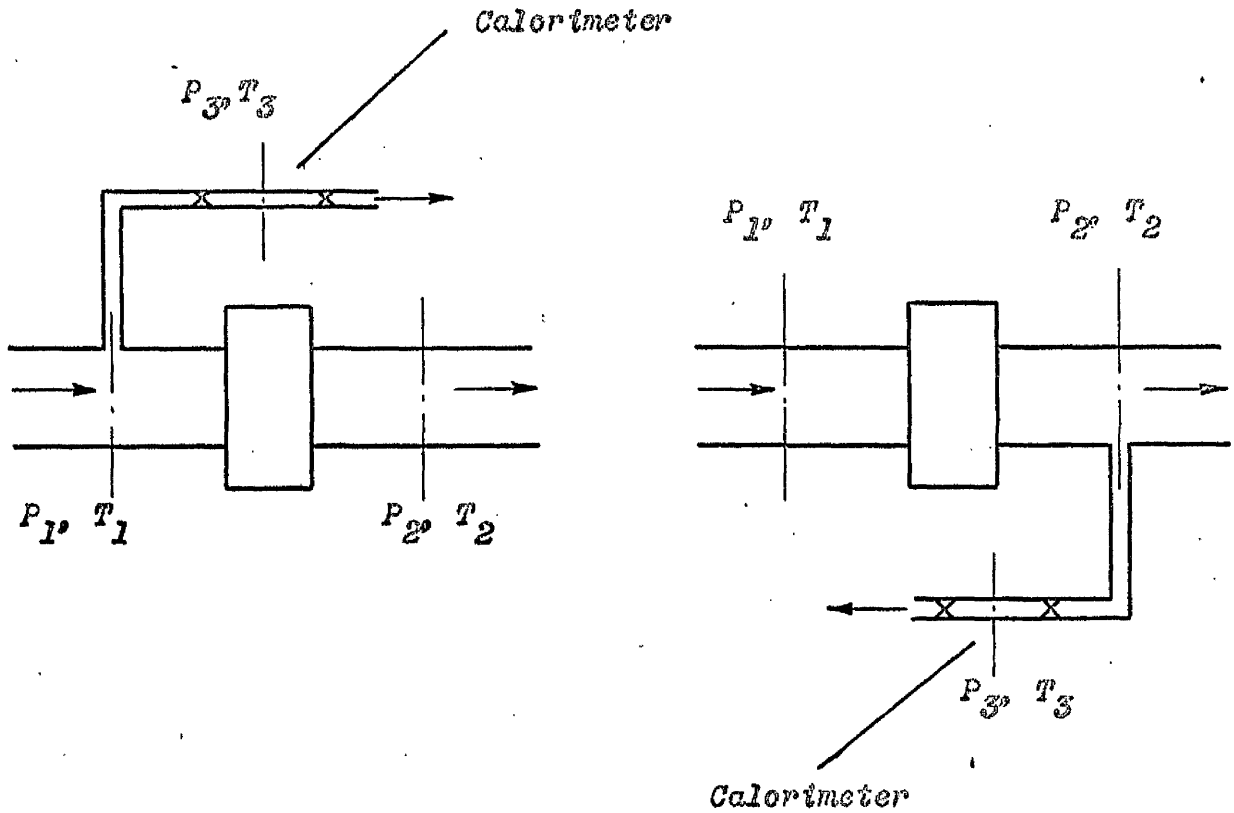
T_R = 18 - 21 °C Av. bridge constant = 3.5 m C° Av. Bridge constant = 5.5 m C° T_R = 23 - 23.5 °C.

TEST NO.	PUMP TEST (CONSTANT SPEED TEST)					TURBINE TEST (CONSTANT RATE OF DISCHARGE)				
	T_1 °C	TOTAL HEAD $H + H_v$ ft.	Q/Q max	CORRECTED ΔT $(T_2 - T_1)$ m C°	+ THERMOMETRIC η_p % — CONVENTIONAL η_p %	T_1 °C	N/N max.	TOTAL HEAD $H + H_v$ ft.	CORRECTED ΔT $(T_2 - T_1)$	+ THERMOMETRIC η_p % — CONVENTIONAL η_p %
1	21.1	114.0	1.00	18.0	~~-0.8~~ +0.7	22.3	0.73	99.0	3.5	~~+2.6~~ +1.5*
2	21.3	118.4	0.98	17.2	~~+0.8~~ +2.0	22.5	0.76	99.0	3.6	~~+1.4~~ +0.2
3	21.4	122.2	0.94	18.4	~~-1.8~~ -0.7	22.3	0.80	98.6	3.4	+0.7 -0.2
4	21.4	124.5	0.91	21.5	~~-3.1~~ -2.3	22.3	0.83	102.4	2.3	+2.5 +1.5*
5	21.5	127.0	0.89	21.8	~~-3.2~~ -2.6	22.3	0.86	100.0	3.6	+0.5 -0.2
6	21.8	130.3	0.86	19.2	~~-1.7~~ -1.3	22.4	0.89	100.8	3.8	~~+1.3~~ +0.7
7	21.8	133.6	0.83	20.9	~~-2.1~~ -1.8	22.6	1.00	102.4	3.3	~~-1.3~~ -1.7
8	21.9	136.8	0.80	22.2	~~-2.3~~ -2.3					

During the pump test the detector behaved erratically

Detector was adjusted after turbine test No. 1 above.

* Correction according to equation (10.12) and Tables (10.1) and (10.2).



(a) Turbine

(b) Pump

FIG. 11.1 Schematic arrangement for partial thermometric method.

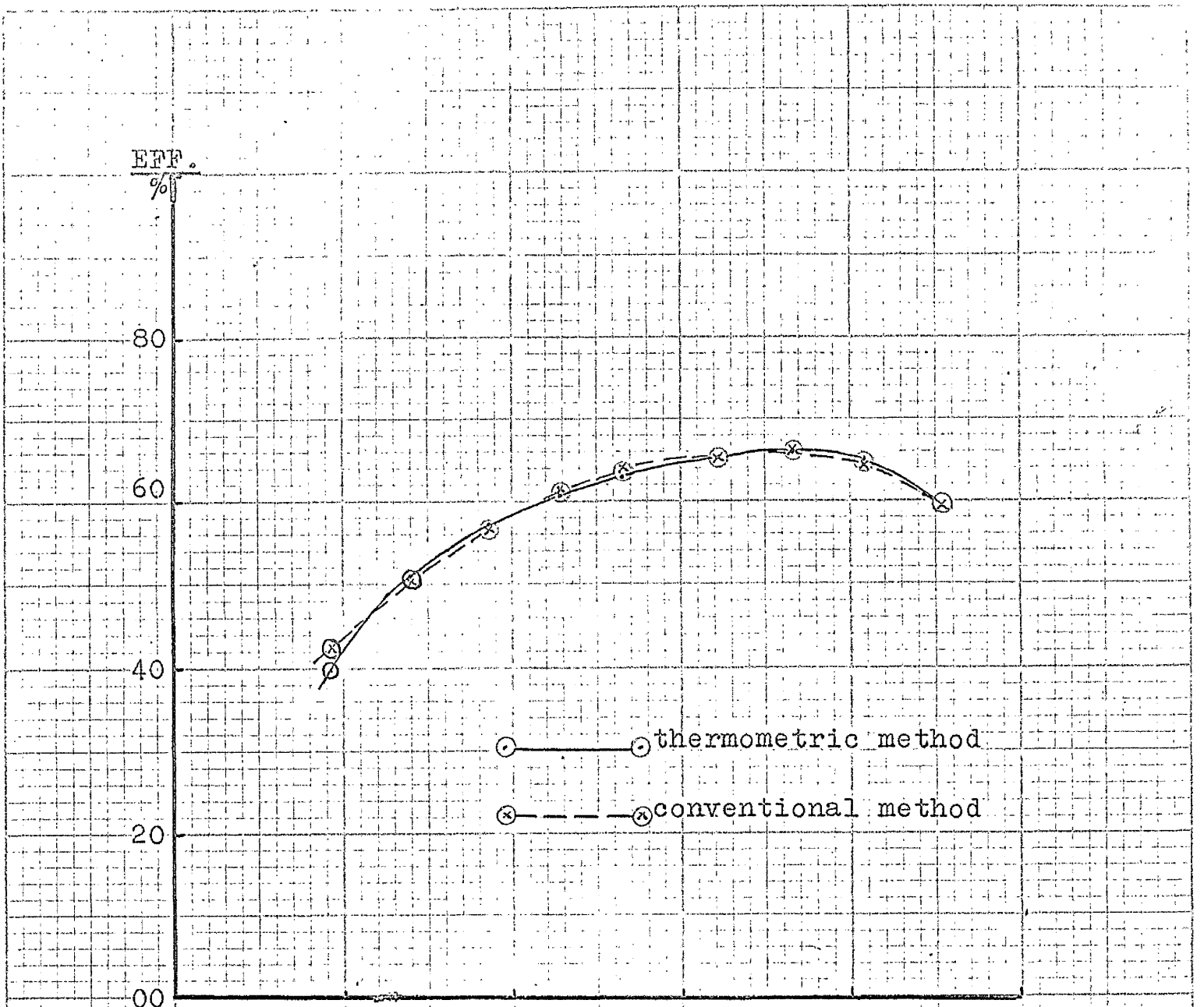


FIG.II.2

EFFICIENCY CURVES: TEST NO.I

ON 4"/3" PUMP

(a.c. bridge)

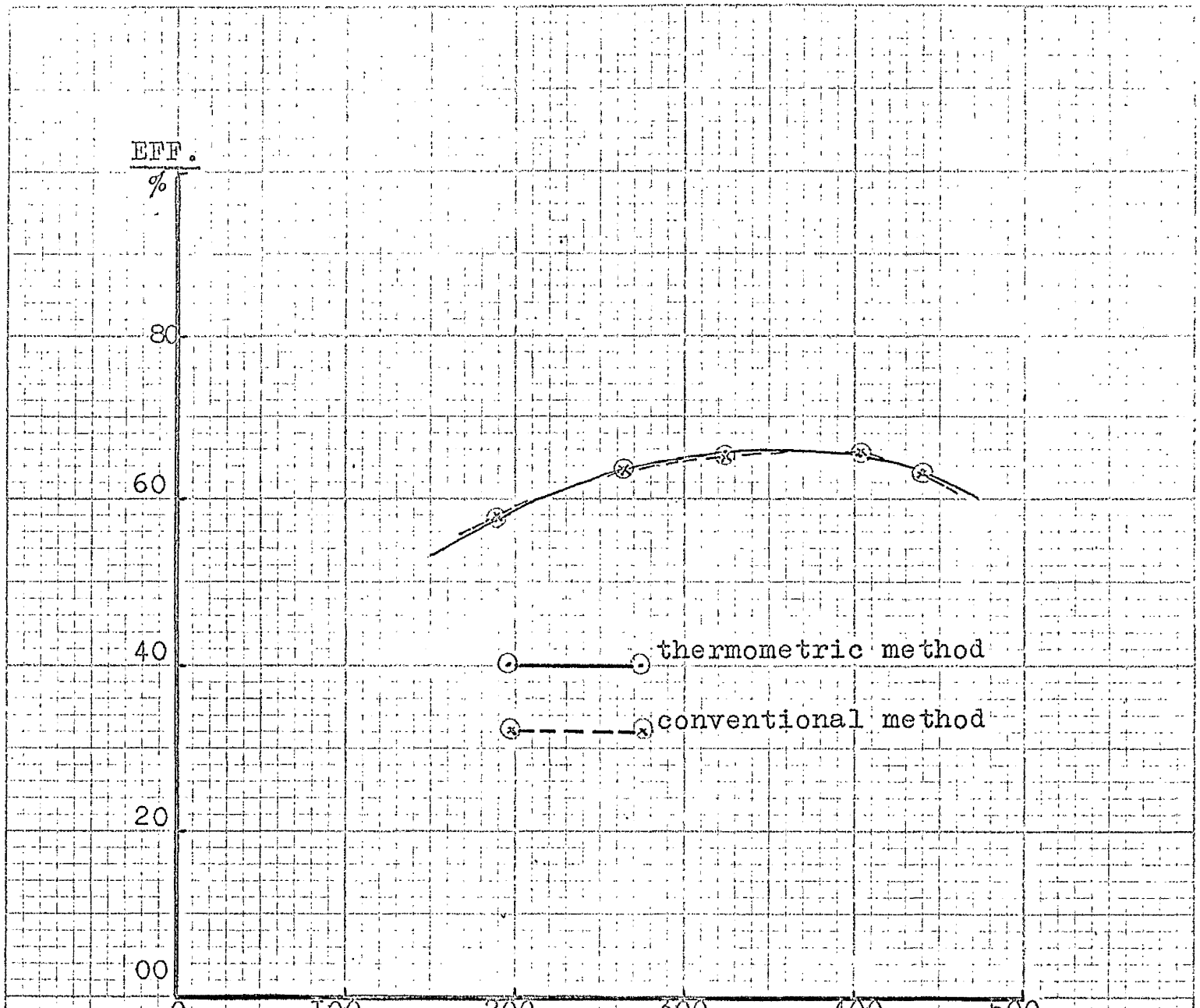


FIG.II.3.

EFFICIENCY CURVES: TEST NO.2

ON THERMALLY INSULATED 4"/3" PUMP

(a.c. bridge)

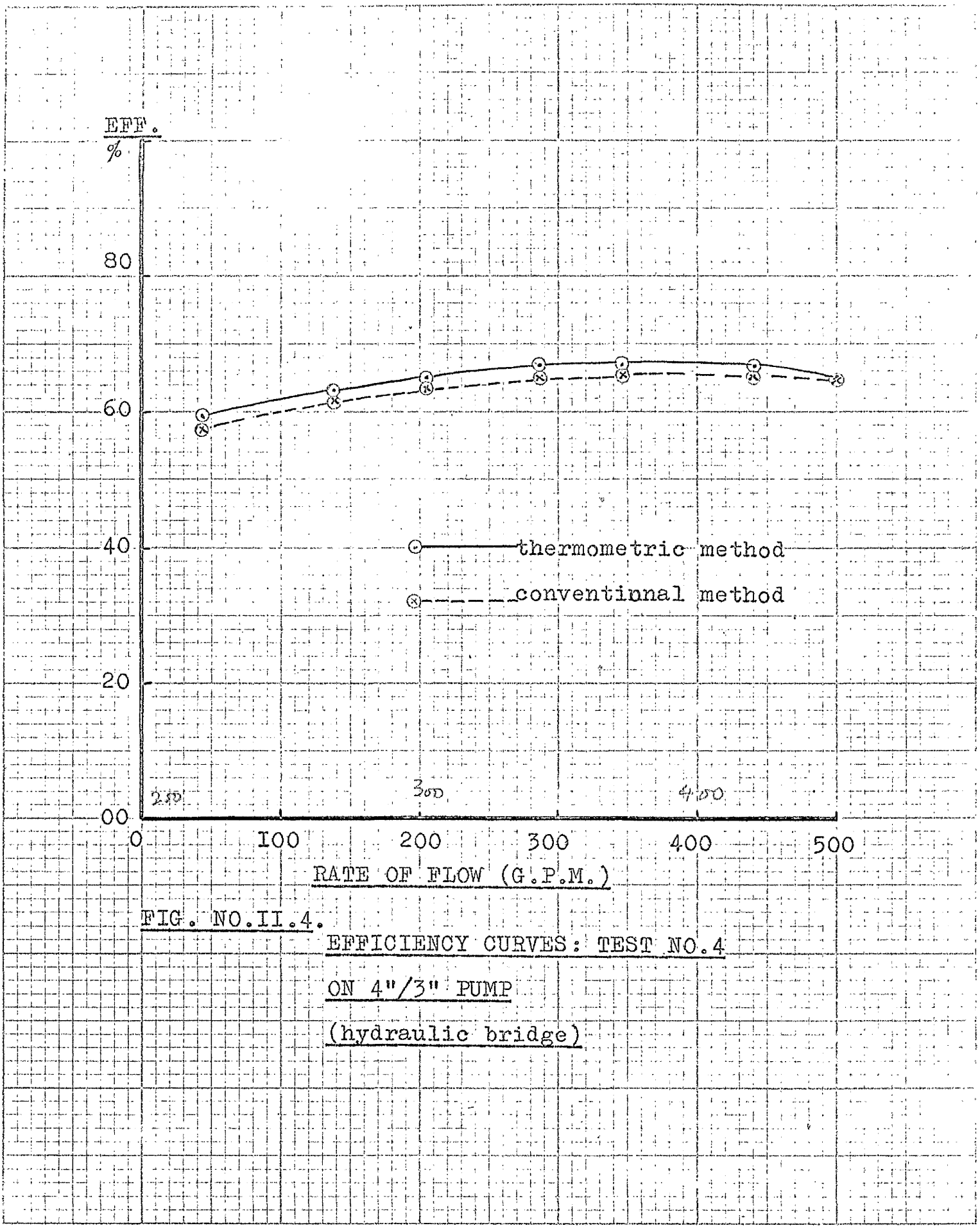


FIG. NO. II.4.

EFFICIENCY CURVES: TEST NO. 4

ON 4"/3" PUMP

(hydraulic bridge)

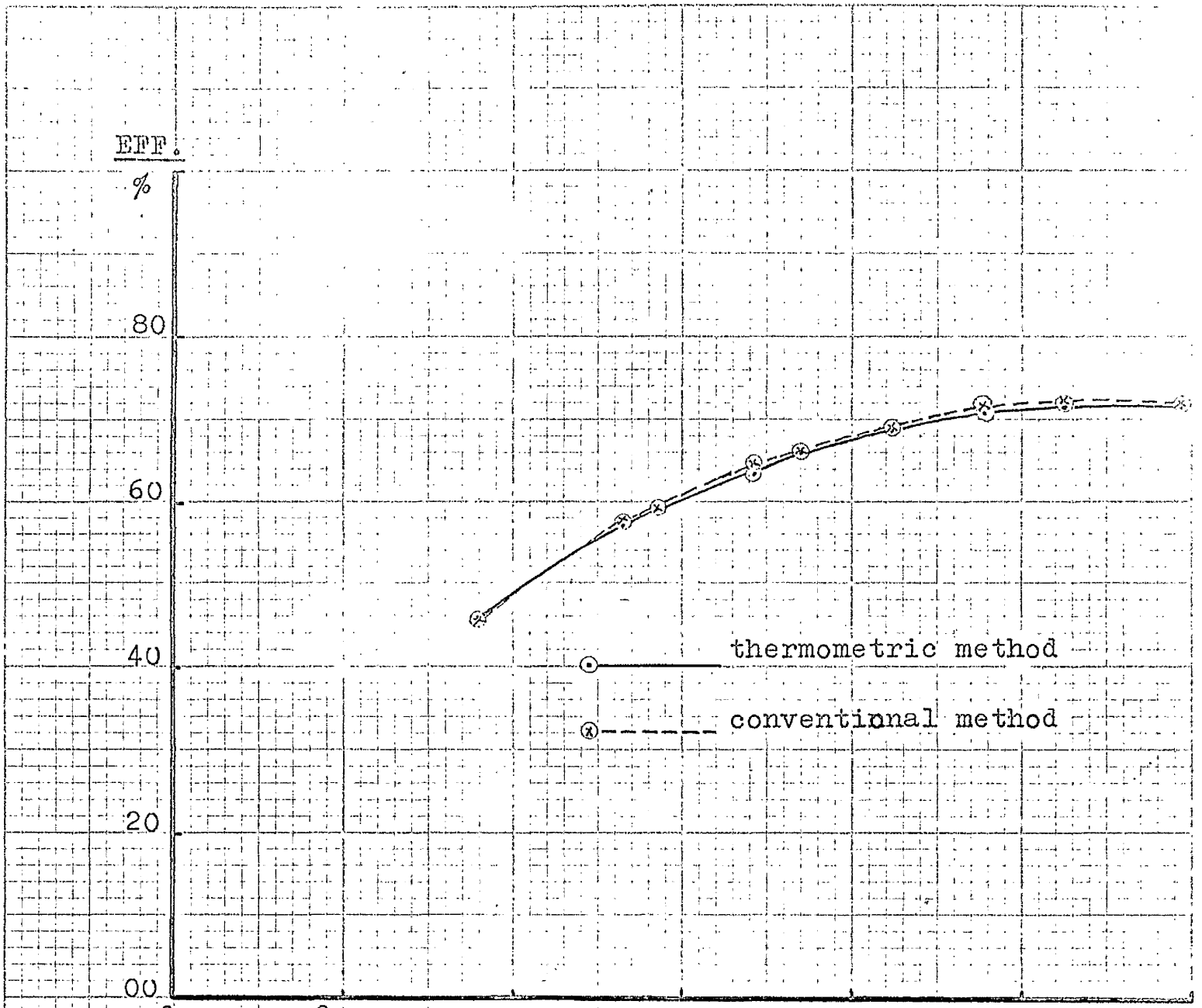


FIG.NO.II.5

EFFICIENCY CURVES: TEST NO.5

ON $2\frac{1}{2}$"/$2\frac{1}{2}$" PUMP

(a.c.bridge)

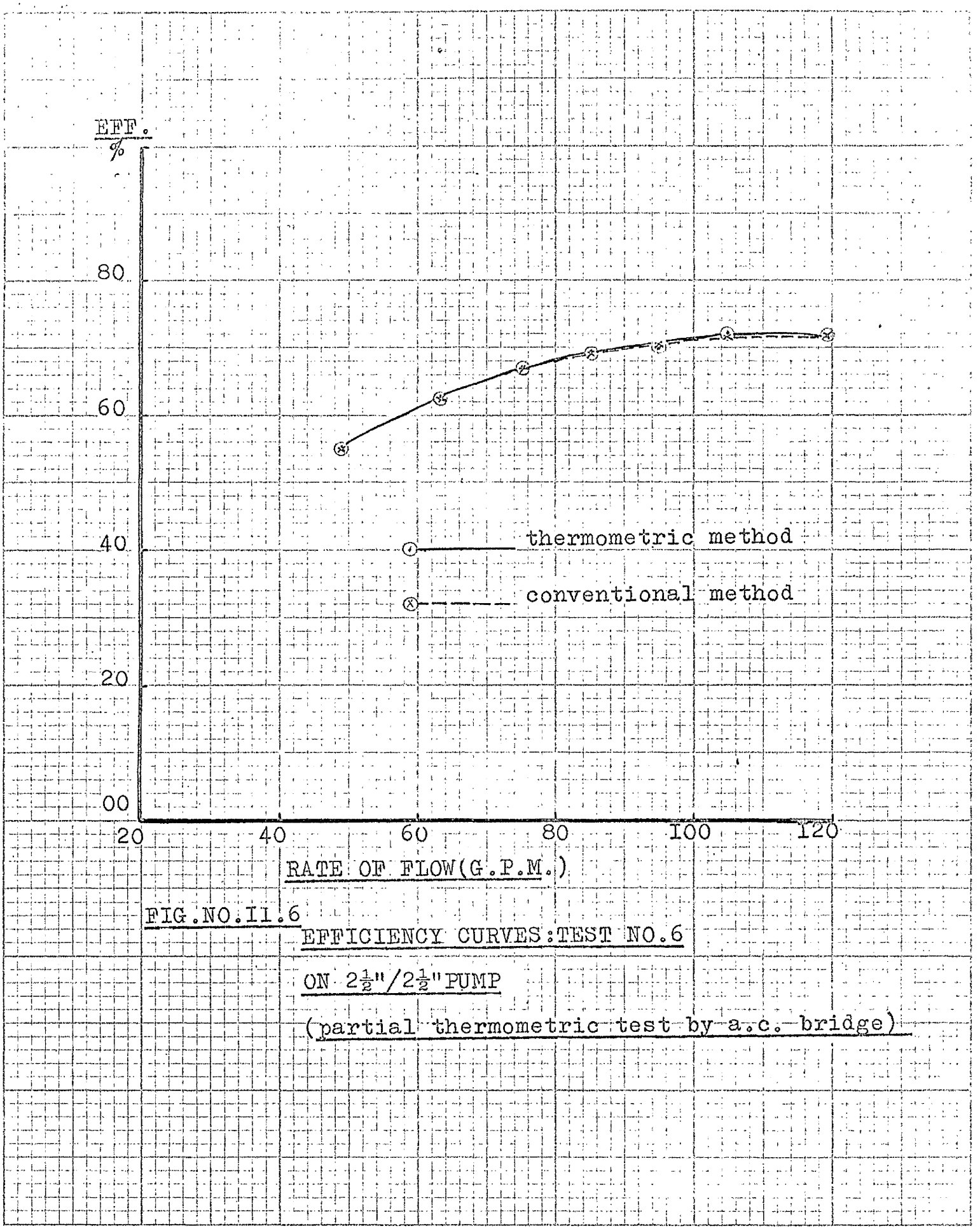


FIG.NO.II.6

EFFICIENCY CURVES:TEST NO.6

ON $2\frac{1}{2}$"/$2\frac{1}{2}$"PUMP

(partial thermometric test by a.c. bridge)

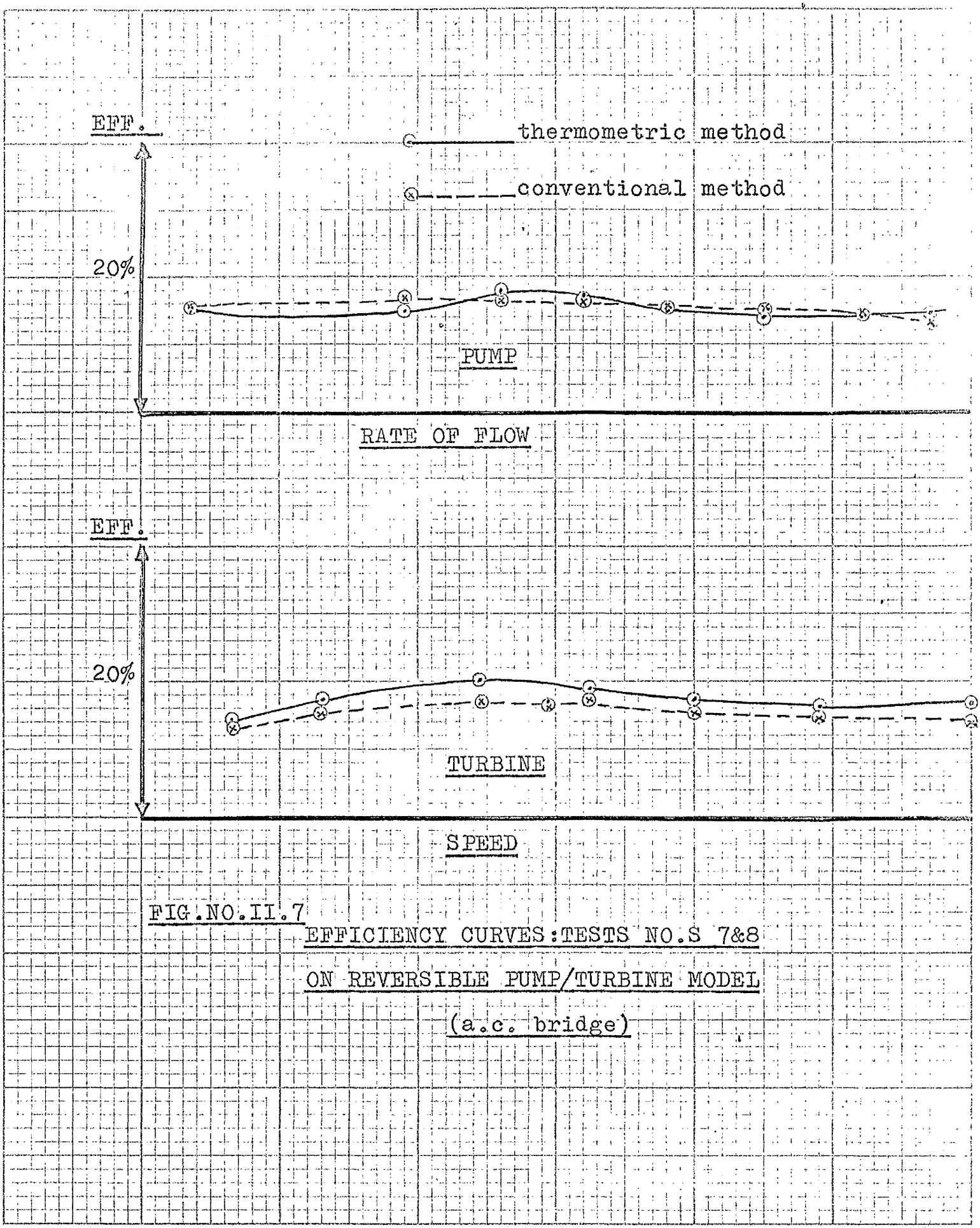


FIG.NO.II.7

EFFICIENCY CURVES:TESTS NO.S 7&8

ON REVERSIBLE PUMP/TURBINE MODEL

(a.c. bridge)

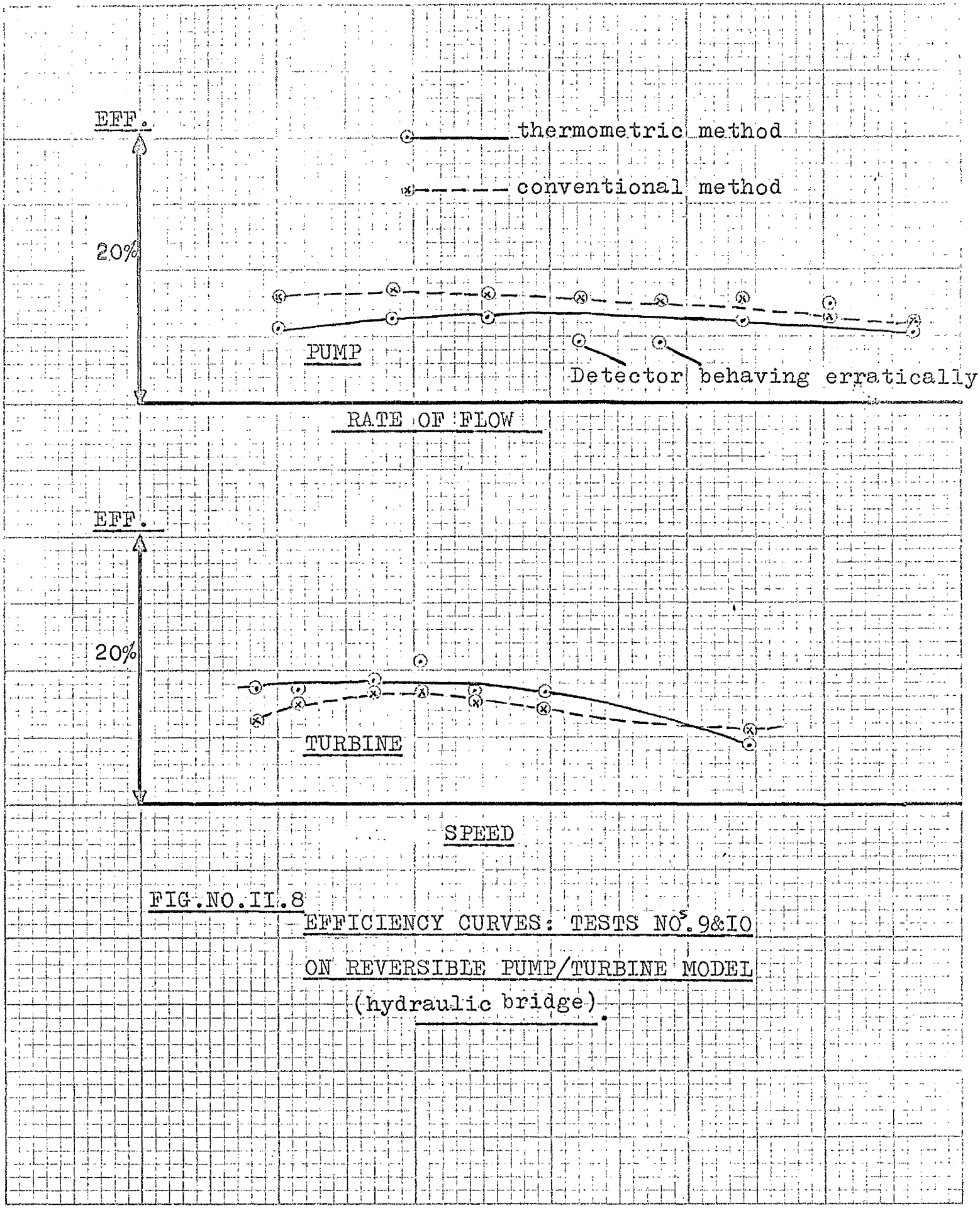


FIG.NO.II.8

EFFICIENCY CURVES: TESTS NO^{S}.9&10

ON REVERSIBLE PUMP/TURBINE MODEL

(hydraulic bridge).

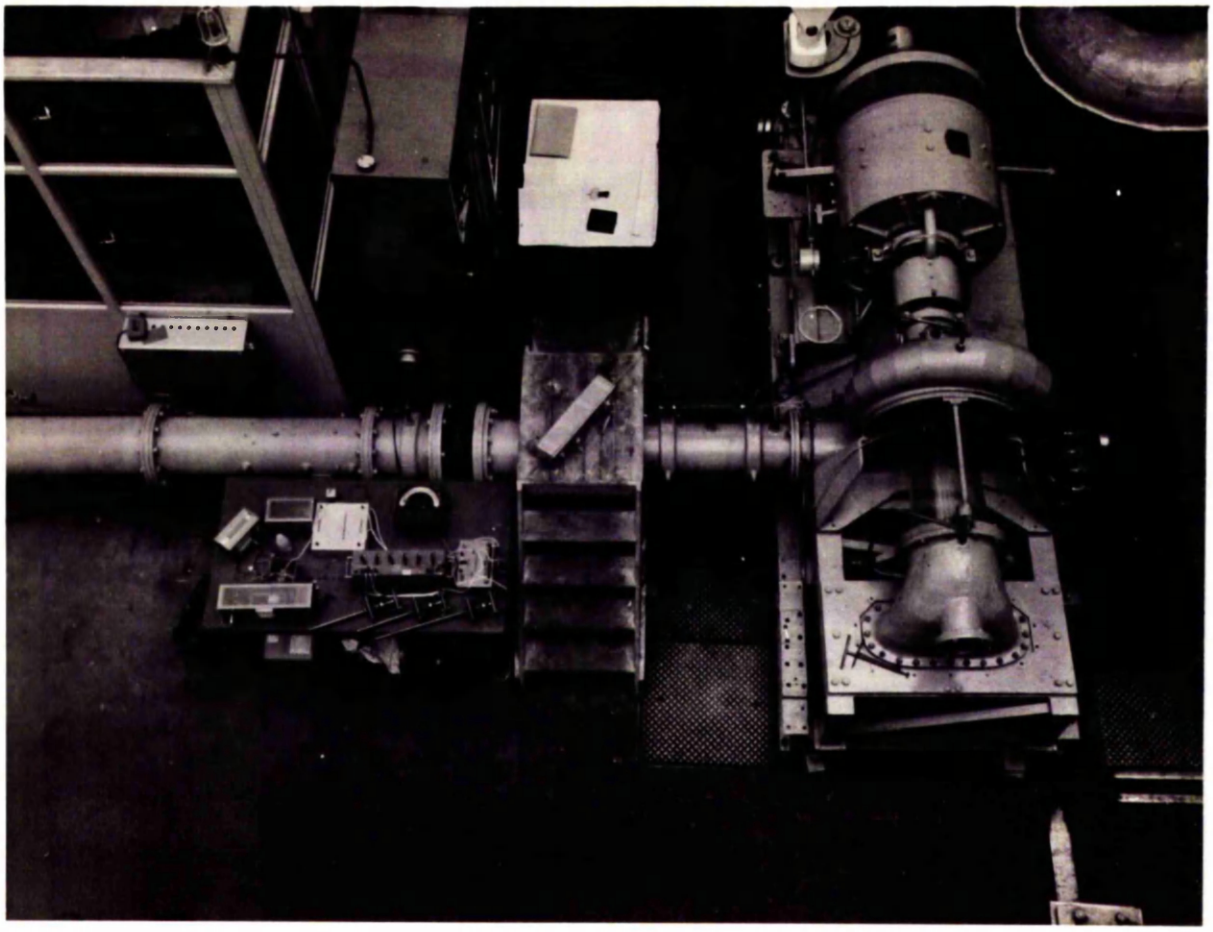


Fig. No. 11.9

The Reversible Pump/Turbine Model

(Note the a.c. transformer ratio-bridge and the thermometer-pockets at the front of the picture)

(by courtesy of Messrs. English Electric Co. Ltd.)

CHAPTER TWELVE

THERMOMETRIC METHODS OF MEASURING THE RATE OF MASS FLOW.

12.1 Introduction

By measurement of the unit-mass and the unit-time rates of exchange of energy between a flowing fluid and a heating or actuating source, the rate of mass flow can be directly determined provided that certain thermodynamic properties of the fluid are known.

The development of the special methods of measuring small temperature differences described in Chapters 7 and 9 renders possible the application of such technique to high rates of flow with very satisfactory results.

The methods of measuring the rate of flow described in this Chapter may be classified in accordance with the type of actuating source used, as following:-

(a) The rotodynamic method,

and (b) The heat diffusion method.

The latter may be classified further as

(i) The ohmic method, (ii) the heat exchange method.

12.2 The rotodynamic method

This method may be used when the flow is actuated by a rotodynamic machine, e.g., a turbine or a pump. The method requires an accurate estimate of the unit-time rate of external work, W (i.e. external power). In the case of turbines this is done by adding all the generator losses to the net generator output. In the case of a pump the motor losses are subtracted from the motor input.

Q_m, the rate of mass flow is obtained from the relation

$$Q_m = W/w \qquad (12.1)$$

where w is the external work done by the fluid per unit mass and is obtained from equation (4.4), the steady flow energy equation,

$$w = A_{T_1}(P_1-P_2) + C_{P_2}(T_1-T_2) + (V_1^2-V_2^2)/2g_u+(Z_1-Z_2)(g/g_u) \qquad (12.2)$$

provided that the thermodynamic properties of the fluid, A_T and C_P are known.

This method was first initiated, and used for turbines by Thom (Ref. 27). However, his exposition of the theory was slightly different from the above since he determined the value of the external work by the efficiency equation instead of by the

steady flow energy equation, e.g.

$$w = \eta_t \times H_T \times (g/g_u) \qquad (12.3)$$

The value of η_t being predetermined by the thermodynamic null method (Section 11.1).

As a demonstration of the practicability of this method of flow measurements, it was applied to observations obtained on the N.E.L. 1155 - ft. pump described in Section (11.4). The same observations shown in Table (11. 6) were used in equations (12.1) and (12.2). The values of the rate of flow so obtained are shown in Table (12.1) together with those determined by weighing. The table shows very close agreement between the two sets of readings ($\pm < 1\%$).

12.3 The heat diffusion methods (Theory)

(i) The ohmic method

For this method an electric immersion heater is inserted in the flow as shown in Fig. (12.1). The principle of the method is the same as that of the experiment based on the constant change of enthalpy described in Section (5.2). If the heat transfer to the surroundings is made negligible by adequate thermal insulation, the total heat diffused into the fluid, Q_e, will be equal to the electric power

input to the heater.

$$Q_e = Q_m \times q \qquad (12.4)$$

$$= \text{Watts} \times 550/746 \quad \text{ft. lb.}_f/\text{sec.}$$

From equations (5.4) and (4.2),

$$q = h_2 - h_1 \qquad (V_1=V_2 \text{ and } Z_1 = Z_2)$$

$$= A_{T_1}(P_2-P_1) + C_{P_2}(T_2-T_1) \qquad (12.5)$$

Thus equation (12.4) may be recast to give an expression for Q_m as following.

$$Q_m = Q_e / \left[A_{T_1}(P_2-P_1) + C_{P_2}(T_2-T_1)\right] \qquad (12.6)$$

(ii) <u>The heat exchange method</u>

For this method a coil of small tubing is installed inside the main pipe (Fig. 12.2). The small tube carries water of temperature substantially different (colder or warmer) than that in the main pipe. From equation (12.4),

for the main pipe $Q_e = q \times Q_m$ (12.7)

for the auxiliary pipe $Q_e' = q' \times Q_m$ (12.8)

Assuming negligible heat transfer to the surroundings of the pipe:

$$Q_e' = -Q_e$$

therefore $$Q_m = -Q_m' \times (q'/q) \qquad (12.9)$$

By substituting for q and q' from equation (12.5), equation (12.9) becomes:

$$Q_m = \frac{(A_{T_1})'(P_1'-P_2')+(C_{P_2})'(T_1'-T_2')}{A_{T_1}(P_2-P_1)+C_P(T_2-T_1)} Q_m' \quad (12.10)$$

(The apostrophe sign indicates that the term refers to conditions in the auxiliary pipe). If the friction losses in the pipes are negligible i.e. $\Delta P = \Delta P' = 0$ and if $(C_p)' = C_p$, then the rates of mass flow in the main and the auxiliary pipes are inversely proportional to the temperature difference:

$$Q_m/Q_m' = |\Delta T' / \Delta T| \quad (12.11)$$

By making Q_m' exceedingly small, and T_1' very different in comparison with Q_m and T_1 respectively, the difference of temperature $(T_1' - T_2')$ will be large and can be measured with an ordinary mercury-in-glass thermometers. Moreover the rate of flow Q_m' in the auxiliary pipe may be determined simply by collecting the discharge in a portable container.

12.4 The heat diffusion methods (Practical experiments)

Practical tests of the above principles were conducted in the Fluid Mechanics Laboratory of

the University of Glasgow as following.

(i) The ohmic method

A 1250-W electric immersion heater was fitted in a 5"-pipe as shown in Fig. (12.1). The measurements of temperature and pressure differences across the heater were made by the hydraulic bridge and a mercury manometer respectively. The electrical power input to the heater was measured by a single phase electro-dynamic Wattmeter with an accuracy of the order of 1.25%.

For comparison, the rate of mass flow was measured by weighing. Table (12.2) shows that the results of the two methods agree to within ± 1.7%.

(ii) The heat exchange method

A radiator consisting of a coil of 3/8" copper tube was fitted inside the 5" - pipe as shown in Fig. (12.2). The tube was connected to the hot water mains. The flow was set to a convenient level so that the temperature drop, $(T_1'-T_2')$ was of the order of 11 - 15 ^{o}C. Mercury-in-glass thermometers with 1/10 ^{o}C graduations were used. The discharge from the copper tube was collected in a 2-gallon container and the rate of flow, Q_m' was determined by

weighing. The pressure difference across the radiator in the main pipe was measured by a mercury manometer, but that across the auxiliary tube was found to be negligible because of the low rate of flow. The temperature difference was measured by (a) the hydraulic bridge and (b) the a.c. bridge and using the pair ratio method (Chapter 7). The rate of flow in the main pipe was also measured by weighing. The results obtained are shown in Tables (12.3.a) and (12.3.b). The average error of the thermometric measurements was of the order of 1.5 - 2.0%.

DISCUSSIONS AND CONCLUSIONS

12.5 Limitations of the thermometric method of measuring the rate of flow

In the case of both heat diffusion methods two main precautions have to be taken:

(i) The heating element or tube must be carefully insulated (both thermally and electrically) from the surroundings, e.g. in the case of the ohmic method the immersion element may be fitted on to a 'Tufnol' flange as shown in Fig. (12.1). Thermal insulation of the main pipe was found unnecessary in the case of all the tests described above.

(ii) The downstream temperature measuring points must be at an adequate distance from the heating source to allow thorough mixing of the heated fluid. A distance of 12 - 15 diameters was found appropriate.

In addition it is necessary to ensure beforehand that the surface temperature of the heat or radiator is not detrimental to the chemical stability of the fluid.

To estimate the amount of electrical power or heat necessary for a certain accuracy of flow measurements, $(\delta Q_m/Q_m)$, the following equation may be used.

$$W \text{ or } Q_e = \left| C_p \, \delta(\Delta T) Q_m / (\delta Q_m/Q_m) \right| \qquad (12.12)$$

Thus if the accuracy of the temperature difference measurements, $\delta(\Delta T)$ is 0.001 ^{o}C, then to obtain 1% accuracy in the measurement of a flow of 100 gpm, an input power of about 3160 Watts, 4.25 HP or 1.67 CHU/second is necessary. (It must be noted, however, that this power consumption is only intermittent, because the heater does not need to be left on all the time.)

Equation (12.12) shows that the running cost of the heat diffusion methods is inversely proportional

to the accuracy of the temperature measurement and directly proportional to the rate of flow and to the required accuracy of the flow measurements. Application of equation (12.12) to the results in Tables (12.2) & (12.3) shows that the accuracy of the temperature measurement was of the order of $\pm \frac{1}{2}$ m C^{o}.

In the case of the rotodynamic methods no extra running cost is involved at all. From equations (12.2), (12.6) and (12.10), it is clear that the methods of flow measurement described in this Chapter require knowledge of the thermodynamic properties of the fluid A_T and C_p and the pressure and temperature differences across the actuating medium. If, however, in the case of the heat diffusion methods the frictional losses (and hence the pressure difference) across the heater or radiator is negligible, the need for knowledge of A_T and for the pressure measurement is obviated.

This need can also be avoided by determining the net temperature difference, (q/C_p), due to external heating or cooling alone i.e. exclusive of the heating due to friction in the pipe.

$$\text{Net } \Delta T = \text{Total } \Delta T - (A_T/C_p)(P_1-P_2) \quad (12.13)$$

This may be done by balancing the bridge with the heater switched off and then switched on alternately. The net ΔT is equal to the difference of the two values of ΔT so obtained. The value of 'q' to be used in equation (12.4) and (12.9) is then determined from the relation, net $\Delta T = q/C_p$.

If ratio-bridges are used, the readings of the bridge at the switch-off and switch-on conditions may be regarded as the pair ratio and the test ratio readings respectively. These are then used in the appropriate equations in Chapter (7) to obtain the net ΔT. If a hydraulic bridge is used the switch-off reading is regarded as the 'C' reading of the bridge) or the bridge constant (Chapter 9).

This procedure for avoiding the pressure measurements and the need for knowledge of A_T requires allowing enough time between the observations for the successive heating and cooling of the system and the temperature probes. The use of a more accurate wattmeter or a potentiometer might have improved the accuracy of the test by the ohmic method. However an accurate and consequently expensive arrangement for the measurement of the electrical input is not always warranted unless the accuracy of temperature measurement is of the same order.

In the tests by the heat exchange method the absolute temperature of the mains was fluctuating but the temperature difference was constant to within $\pm$ 1/10 ^{o}C (equivalent to about $\pm$ 1% error in Q_m). Readings were only taken when the fluctuation of the temperature difference was within this limit.

It was recently understood that a method similar to the ohmic method described above was used by Bensemen (Ref. 28) for detecting and/or measurement of rates of flow of water of the order of 0.1 cubic centimetre per day in underground rock. Bensemen used thermocouples which were previously calibrated against known rates of flow. The ohmic method described here is based on the thermodynamic equation of the flow and is absolute and requires no calibration. Furthermore, the techniques of precise measurement of temperature difference described in Chapters (7) and (9) makes it possible to use these thermometric methods with much higher rates of flow than those tested by Bensemen.

12.6 Advantages of the thermometric methods of measuring the rate of flow

(i) They are absolute methods, in this they are directly comparable with the weighing tank, as all other

flow measuring devices, e.g. orifices, venturi-meters and anemometers are indirect and only indicate velocities. Furthermore the thermometric methods can be used successfully in closed circuits where a weighing tank can not be used.

(ii) The equipment needed for the thermometric methods is cheap, flexible and portable. The head losses entailed by permanent installation of a heater or a small radiator tube in a pipe is likely to be negligible compared with that due to an orifice or venturi-meter. Thus a number of heaters may be installed at different points in the Laboratory. This is in contrast to the case of weighing tanks or venturi-meter which necessitate designing all laboratory piplines so as to converge into one main pipe and hence restricting the flexibility of the laboratory.

(iii) Although the accuracy of the thermometric methods in many cases may not be comparable with that of the other conventional methods and at the same time have equally economical running cost, neverthless they may provide a promising solution for the problem of measurement of the rate of flow of liquids which are difficult to handle, e.g. hot water in feed-water pumps. Here the heat exchange

method may provide a simple solution, because water at ambient temperatures may be used in the radiator. The thermometric methods can also be used for remote reading of the rate of flow.

12.7 CONCLUSIONS

The results of the preliminary tests of the principles of the thermometric methods described here are very promising and warrant more comprehensive investigations. Further improvement of the techniques of measuring temperature and of the robustness and accuracy of the temperature measuring apparatus may well render practicable the application of these methods under field conditions and even to higher rates of flow.

FIGS. 12.1 and 12.2 HEAT DIFFUSION METHODS FOR MEASURING RATE OF FLOW

IMMERSION HEATER MOUNTING

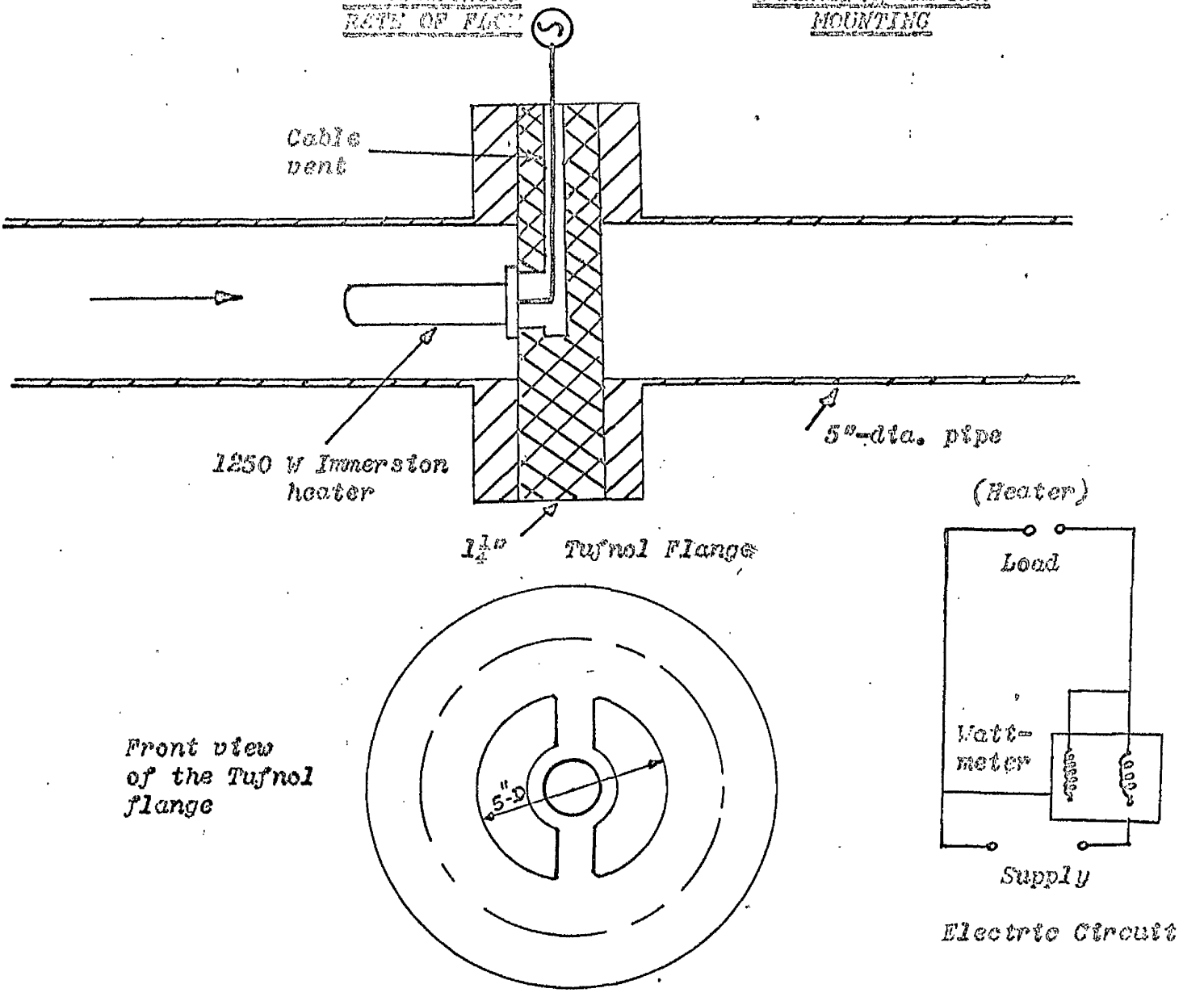


FIG. 12.1 IMMERSION HEATER: CIRCUIT AND MOUNTING

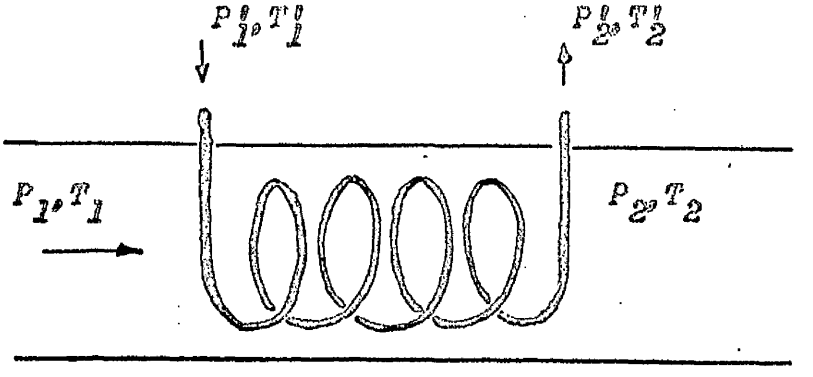


FIG. 12.2 RADIATOR TUBE

TABLE NO. 12.1 *

THERMOMETRIC METHODS OF MEASURING THE RATE OF FLOW (ROTODYNAMIC METHOD)

RESULTS OF TEST on 1155 FT. PUMP

Test No.	Rate of Flow (by weighing)		Rate of Flow (Thermometric)	
	gpm	lb_m/sec.	gpm	lb_m/sec.
1	35.8	5.97	35.8	5.97
2	53.0	8.84	52.2	8.70
3	56.5	9.42	56.4	9.40
4	68.2	11.37	67.1	11.19
5	74.0	12.33	73.8	12.30
6	84.5	14.09	84.0	14.00
7	95.6	15.94	94.3	15.72
8	105.2	17.54	104.2	17.37
9	119.0	19.84	118.7	19.79

Average Error = $\pm$ 0.9%

* From N.E.L. report No. 130 by Foord, Langlands & Elagib.

TABLE NO. 12.2

THERMOMETRIC METHODS OF MEASURING THE RATE OF FLOW

heat diffusion methods

(i) The ohmic method

Test No.	Heater input Watts	Net ΔT due to external heating (q/C_p)	Rate of flow (Thermometric)		Rate of flow (Weighing)	
			lb_m/sec.	gal/min	lb_m/sec.	gal/min.
1	1248	32.5	20.3	122	20.0	120.2
2	1242	27.2	24.0	144	24.2	145.0
3	1252	24.0	27.5	165	27.1	162.5
4	1288	22.2	30.5	183	30.0	180.0
5	1272	18.6	36.0	216	35.4	212.2
6	1252	17.5	37.7	226	37.0	222.1
7	1220	15.1	42.5	255	43.5	260.7

Average error + 1.7%

The head loss due to the fitting of the heater in the pipe was of the order of 4%.

TABLE NO. (12.3). THERMOMETRIC METHODS OF MEASURING THE RATE OF FLOW

HEAT DIFFUSION METHODS

(11) THE HEAT EXCHANGE METHOD

TABLE NO. (12.3.a) RESULTS OF TESTS BY THE HYDRAULIC BRIDGE

Test No.	Main pipe		Radiator tube		Rate of flow	Rate of flow	
	Total ΔT* m C ^{o}	Net ΔT m C ^{o}	Net $\Delta T'$ deg. C ^{o}	Rate of flow lb$_m$ / min.	(Thermometric) lb$_m$ / sec.	(Weighing) lb m/sec.	gal/min.
1	28.7	22.6	11.8	1.51	13.2	13.7	82
2	28.2	20.2	12.2	1.51	15.2	15.7	94
3	29.8	19.1	10.6	1.94	18.0	18.2	109
4	32.2	18.8	11.8	1.98	20.7	20.3	122
5	34.4	16.9	12.4	1.85	22.7	23.2	139
6	35.3	16.3	12.6	1.85	23.8	24.2	145
7	37.0	11.5	13.0	1.51	28.5	28.0	168

$\bar{T}_1$ ≒ 16 ^{o}C T'_1 (hot water) ≒ 40 - 44 ^{o}C T_R ≒ 18 - 19 ^{o}C Average error ± 2.0%

TABLE NO. (12.3.b) RESULTS OF TESTS BY THE A.C. BRIDGE

Test No.	Main pipe		Radiator tube		Rate of flow	Rate of flow	
	Total ΔT* m C ^{o}	Net ΔT m C ^{o}	Net $\Delta T'$ deg. C ^{o}	Rate of flow lb$_m$ / min.	(Thermometric) lb$_m$ / sec.	lb$_m$ /sec.	gal/min.
1	32.5	22.3	12.3	1.93	17.7	17.8	107
2	32.5	18.8	12.0	1.93	20.5	20.8	125
3	33.0	16.3	11.6	1.93	22.8	22.8	137
4	38.0	11.7	13.6	1.93	27.8	28.3	170
5	39.6	15.8	13.5	1.93	27.5	27.3	164
6	57.8	13.0	14.4	1.93	35.7	36.5	219

$\bar{T}_1$ ≒ 14 ^{o} C T'_1 (hot water) ≒ 40 - 44 ^{o}C T_R ≒ 19 - 20 ^{o} C Average error ± 1.5%

(Net ΔT = ΔT due to external heating = q/C_p)

* The head loss due to installation of the radiator tube was about 2 %. The high head loss shown in the table was due to an orifice-meter fitted between the pressure measuring points.

CHAPTER THIRTEEN

CONCLUSIONS

1. Use of Bernoulli (and Euler) equations in Hydraulics is restricted to flows which can be considered frictionless and in which there is no external work transfer. This is often a serious limitation.

Flow of a liquid causes changes in the thermodynamic state of the liquid - these changes of state being inevitably irreversible. It is clear therefore that a complete description of liquid flow processes can only be obtained by using the more general equations of Thermodynamics. The main obstacle to this procedure has been the difficulty of measuring the small temperature differences encountered in liquid flow and on which the thermodynamic equations are dependent.

2. The development of the transformer ratio-bridge (Ref. 7), the pair ratio method and the hydraulic bridge (Chapters 7 and 9) reduced substantially this difficulty of temperature measurement. By the use of these instruments

and methods, it is in most cases possible to obtain an accuracy of about $\pm$ 1 mC^{o} in the measurement of temperature difference. Under favourable conditions an average accuracy of $\pm$ 0.3 mC^{o} has been attained.

3. A suitable equation for use in "Thermohydraulics" or the thermodynamic treatment of liquid flow, is the version of the Steady Flow Energy Equation evolved in Chapter 4. This equation makes possible the estimation of the results of any thermodynamic process taking place in the flow system, directly from the thermodynamic properties of the fluid. In contrast with the Bernoulli and Euler' equations this equation depends on both the temperature and pressure as variables. The equation has been used in the analysis of a number of hydraulic flow systems of varied nature with very satisfactory results.

4. It was proved both theoretically and practically that heat transfer from the surroundings has negligible effects on measurements provided that the fluid is at, or close to,

the ambient temperature. A simple equation for estimation of these effects was presented (Chapter 6).

5. It was found that it is necessary to make corrections for the error resulting from stagnation and/or frictional heating of the thermometer pockets when they are fitted in pipes of different cross-sectional area. Two methods of correction were developed (Chapter 10).

6. The use of a hydraulic throttling calorimeter for calibration and standardisation of temperature measuring apparatus was very successful. It is believed that this device may prove invaluable in any laboratory concerned with thermohydraulic research.

7. It is concluded that platinum resistance thermometers are at present the most suitable type of thermometer for the precision thermometry required in Thermohydraulics. It is noted however that the accuracy of thermometric measurement is in most cases limited by the

instability of the thermometer element (and of the flow) rather than by the insensitivity of the element. From this point it is evident that research is badly needed in the field of thermometer design and construction. Robustness and stability must be the main objectives.

8. The satisfactory results obtained from the efficiency and rate of flow tests described in Chapters 11 and 12 demonstrate clearly the practicability of using temperature as a variable in Hydraulics, and the advantages of the use of the special form of the Steady Flow Energy Equation mentioned above.

The thermometric methods of measuring hydraulic efficiency and rate of flow described in these Chapters are very simple and require inexpensive apparatus and minimum interruption of the operation of the plant.

9. Finally, it is hoped that the methods of precision differential thermometry developed in this research and further advancements of thermometry will inspire more confidence in the use of temperature as a variable in Hydraulics

and stimulate interest in Thermohydraulics

- the thermodynamic treatment of liquid flow.